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Correlation Measures for q-rung Orthopair m-polar Fuzzy Sets with Application to Pattern Recognition

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ABSTRACT

Correlation provides a relationship between two quantities with the help of certain statistical data. It shows the strength of the relationship amongst different alternatives. q-rung orthopair fuzzy setting offers extended space for the selection of values of the two components. There is no iota in the fact that the multipolarity factor reduces the chances of error. We study correlation in the framework of q-rung orthopair m-polar fuzzy set to judge the efficiency of this system in real-life problems and decision-making. We present some major characteristics of the correlation measure in the said setting. An application to pattern recognition is also incorporated. To elaborate on the ideas of correlation measures and q-rung orthopair m-polar fuzzy set, we instigate a comparison analysis to probe the supremacy and dominance of propound abstraction.

1. Introduction

To manage daily life situations involving vagueness, imprecision, and uncertainty, [1] expanded the traditional notions of classical sets to fuzzy sets and initiated the basis of fuzzy mathematics. A fuzzy set allocates partial grades (known as membership degrees) that fall in $[0, 1]$ to convey the extent of an element's membership in a set concerning specific characteristics. The fuzzy set caught the attention of the researchers. In today's world, the notions of fuzzy sets are widely employed in numerous areas inclusive of artificial intelligence, engineering, neural networks, information fusion, computational intelligence, linguistics, pattern recognition, medical analysis, machine learning, and decision-making

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problems.

According to [2], and [3], an intuitionistic fuzzy set (IFS) is an expansion of a classical (traditional) fuzzy set. An IFS is a very beneficial and proficient model that allocates non-membership degrees (NMDs) in addition to membership degrees (MDs) to each member of the underlying set with the restriction that the aggregate of these grades lies within the real interval of unit length(Magnitude, norm). [4] coined the Pythagorean fuzzy set (PFS) notion. PFSs can handle the problems when the aggregate of MD and NMD exceeds unity, but the sum of their squares still falls in $[0, 1]$. The concept of Pythagorean m-polar fuzzy set (P-m-PFS) is introduced by [5]. Further generalized the notion to Pythagorean m-polar fuzzy soft set (P-m-PFSS) [6].

Building on the effectiveness of PFSs, [7] proposed an even more robust model called a q-rung orthopair fuzzy set (q-ROFS) [8]. Utilized GRA and TOPSIS to model uncertainties in COVID-19 using q-rung orthopair m-polar fuzzy soft information. Demonstrated the application of q-rung orthopair m-polar fuzzy sets in multi-criteria decision-making in robotic agri-farming [9]. As the studied of some new score functions for MADM in the framework of generalized orthopair fuzzy membership grades [10]. To analyzed weighted aggregation operators for q-rung orthopair m-polar fuzzy with applications [11]. Making use of using rough mF bipolar soft environment, [12] presented a hybrid decision-making framework. In the framework of q-ROFSs, [13] utilized neutrality aggregation operators for the purpose of multiattribute group decision making. q-rung orthopair fuzzy soft topology for MADM was studied by [14, 15].

Two variables are usually related or associated with each other in some way or to some degree. For instance, the income and expenditures of an individual are related to each other. Likewise, the height and weight of a person depend on each other. In daily life, we observe that a commodity's price change depends upon its supply and demand in the market. Such relationships between two or more objects may be studied using correlation. Correlation measures the interdependence, relationship, or association between two objects, in which changes in one object's values are affected by the changes made in the value of the other object. In statistics, the degree of correlation is measured using the correlation coefficient, r, where $r \in [-1, 1]$. The positive value of r indicates that the two variables are mutually influenced. Two variables are a perfect match if the value of r is 1. If $0 < r < 1$, then two variables are mutually influenced and if $-1 < r < 0$, two variables are inversely related.

In the context of different fuzzy sets expansions, the correlation measure's value, r, lies in $[0, 1]$. Some correlation measures for IFSs were suggested by [16, 17]. Later, [18] proposed correlation measures for PFSs. On the basis of shapley weighted divergence measure, [19] extended IF TODIM technique for MADM. Under Pythagorean fuzzy soft environment, [20] studied similarity measure with utilizations. [21] and [22] pointed out some deficiencies in the former correlations and coined new directional correlations for PFSs. [23] utilized correlation measures to develop an extended TOPSIS method based on the PFS environment. For the mask selection procedure, correlated TOPSIS was explored by [24, 25]. Weighted correlation for MCDM was introduced by [26]. The suggested Pythagorean fuzzy information measures with their utilizations in clinical diagnosis, pattern recognition, and clustering analysis. Information measures for q-rung orthopair fuzzy sets were presented by [27, 28]. The extended TOPSIS method featuring coefficient for MCDM with IFSS information was established by [29]. Recently, unveiled some modified Pythagorean fuzzy correlation measures and their potential applications [30, 31]. As fuzzy theory based complementary concept, [32] coined the conception of knowledge measure and entropy. To initiated improved correlation coefficients for q-ROFS accompanied by their utilizations in cluster analysis. Using picture fuzzy setting, [34] unveiled a new similarity measure along with their practical utility towards clustering analysis and pattern recognition. In recent times, [35, 36] presented fuzzy mean codeword length and similarity measure. Under m-polar neutrosophic environment, [37] studied several information measures for MADM. In recent times, [38] presented practical utilizations towards movie recommender system and robotics by making use of comparison measures for Pythagorean m-polar fuzzy sets. [39, 40] initiated an axiomatically supported divergence measures for q-ROFSs. On the basis of information entropy, [41] studied group decision making under social influences. In Pythagorean fuzzy environment, [42] studied some correlation coefficients. [43, 44] rendered application to multi-criteria decision-making utilizing generalized hesitant fuzzy knowledge measure. A novel entropy and similarity measures in Fermatean fuzzy framework were presented recently by [45] for MADM.

2. Review Of Literature

The concept of correlation has been extensively studied in the context of fuzzy set theory and its various extensions. Correlation measures play a crucial role in quantifying the strength and nature of the relationships between different variables, which is vital for decision-making, pattern recognition, and other real-world applications. In the realm of classical fuzzy sets, [46] proposed several correlation measures for intuitionistic fuzzy sets (IFSs), which extended the notion of correlation to the intuitionistic fuzzy environment [47]. Their work laid the foundation for analyzing the interdependence between IFS components, namely, the membership and non-membership degrees. Building on this, Gerstenkorn and [48] and [49] further explored correlation measures for IFSs and demonstrated their applications in medical diagnostic problems.

In this more expanded fuzzy framework, most of the studies of correlation measures started with the development of Pythagorean fuzzy sets (PFSs) approaches. From all of these the one of the earliest studies to address the shortcomings of previous measures was [50], that proposed correlation coefficients for PFSs that took into account the fact that the total of squares for membership and non-membership degrees could not equal unity. In order to accurately represent the subtleties of the Pythagorean fuzzy environment, [51] and [52] subsequently strongest matches certain shortcomings in the PFS correlation measures that were already in place and incorporated additional directional correlation coefficients [53, 54].

[55] proposed the idea of Pythagorean m-polar fuzzy sets (P-m-PFSs), which expands on PFSs' ability to handle multi-polar data. The recommendation was then expand to Pythagorean m-polar fuzzy soft sets (P-m-PFSS) by [56], who also looked at similarity metrics in this setting. The examination of correlation measures in the multi-polar fuzzy context was made possible by these important development. The concept of q-rung orthopair fuzzy sets (q-ROFSs) was introduced by [57], providing an even more flexible foundation for representing imprecise and uncertain data. Improved correlation coefficients for q-ROFSs were presented and their uses in cluster analysis were illustrated by [60]. Furthermore, [58, 59] investigated axiomatically supported divergence metrics for q-ROFSs, that added to the collection of analytical resources in this specific field.

As noted by [61–64], q-rung orthopair m-polar fuzzy sets (q-ROm-PFSs) are the result of incorporate multi-polarity and q-rung orthopair fuzzy sets. This hybrid construct is a practical framework for correlation analysis since it offers an improved capacity to manage complicated, unpredictable, and also the multifaceted information. Be that as it may, a thorough discussion of correlation measures created especially for q-ROm-PFSs and their real-world applications is lacking in the literature straightaway in the publication [65]. By presenting new correlation and weighted correlation measures for q-ROm-PFSs and providing a comprehensive and detailed analysis of their theoretical characteristics and also their application in pattern recognition and decision-making scenarios, the current study seeks to close this gap. Building on the groundwork established by prompt research on correlation measures in fuzzy and multi-polar fuzzy environments, this work introduces novel methods particularly designed for the q-ROm-PFS framework approaches [66].

3. Methodology

Now the creation of new correlation and weighted correlation measures for q-rung orthopair m-polar fuzzy sets (q-ROm-PFSs) is one of the main focuses of this research project in this paper. The objective of these metrics is to measure the degree and type of relationships that exist among many of the variables or alternatives that are included in the q-ROm-PFS framework. A q-ROm-PFS is characterized by m pairs of membership and non-membership functions that satisfy the condition that the sum of their q-th powers does not exceed unity, where $q > 1$.

The proposed correlation measure between two q-ROm-PFSs A and B captures the covariance between the corresponding membership and non-membership degrees, normalized by the product of the standard deviations. The weighted correlation measure further incorporates the relative importance of different components through assigned weights. These measures range from -1 to 1, indicating perfect negative correlation, no correlation, and perfect positive correlation, respectively. The measures are sensitive to changes in the q-ROm-PFS values and satisfy desirable mathematical properties such as symmetry, boundedness, and normalization [67, 68].

The general procedure involves representing the alternatives or decision criteria as q-ROm-PFSs to apply the proposed correlation measures, calculating the correlation coefficients, and ranking the alternatives based on the obtained correlation values. If necessary, sensitivity analyses can be conducted to examine the robustness of the rankings with respect to changes in the q-value or the component weights. The detailed mathematical formulations, properties, and step-by-step procedures establish a robust methodological framework for utilizing the proposed correlation measures in q-ROm-PFS-based decision-making and pattern recognition problems [69–71].

In several real-life scenarios, we come across uncertain information that is multi-polar, and thus there arises a need to identify the correlation between them. This article aims to provide correlation measures within the framework of q-rung m-polar fuzzy sets to address this issue. The hybrid structure of the q-rung orthopair m-polar fuzzy set is more fruitful and productive in handling imprecision and uncertainty. The insights related to correlation and weighted correlation measures for the q-rung orthopair m-polar fuzzy set have been introduced. For the sake of ranking the feasible choices, we used the score function. Practical utility to pattern recognition is also incorporated here to examine multi-criteria decision-making (MCDM) [72].

The discussion is based on six sections. Section 2 covers the basics of the q-rung orthopair m-polar fuzzy set. Section 3 offers correlation and weighted correlation measures associated with the q-rung orthopair m-polar fuzzy set accompanied by their premier characteristics. Section 4 deals with an application to pattern recognition gleaned from correlation measures for a q-rung orthopair m-polar fuzzy set. Section 5 concludes the article.

4. Preliminaries

This section comprises q-rung orthopair m-polar fuzzy set (q-ROm-PFS) and its imaginative characteristics to work out for the running document.

Definition 1. [66] We consider $\mathfrak{X}$ as a classical set. A well known q-rung orthopair fuzzy set (q-ROFS) with $q > 2$ may be discussed as

$$\mathfrak{R} = \left\{ \langle \chi, \mu_{\mathfrak{R}}(\chi), \nu_{\mathfrak{R}}(\chi) \rangle : \ 0 \leq \mu_{\mathfrak{R}}(\chi)^q + \nu_{\mathfrak{R}}(\chi)^q \leq 1, \chi \in \mathfrak{X} \right\}$$

Table 1
Tabular form of a q-rung orthopair fuzzy set

$\mathfrak{R}$			
χ_1	$\left(\mu_{\mathfrak{R}}^{(1)}(\chi_1), \nu_{\mathfrak{R}}^{(1)}(\chi_1)\right)$	$\ldots$	$\left(\mu_{\mathfrak{R}}^{(m)}(\chi_1), \nu_{\mathfrak{R}}^{(m)}(\chi_1)\right)$
χ_2	$\left(\mu_{\mathfrak{R}}^{(1)}(\chi_2), \nu_{\mathfrak{R}}^{(1)}(\chi_2)\right)$	$\ldots$	$\left(\mu_{\mathfrak{R}}^{(m)}(\chi_2), \nu_{\mathfrak{R}}^{(m)}(\chi_2)\right)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
χ_s	$\left(\mu_{\mathfrak{R}}^{(1)}(\chi_s), \nu_{\mathfrak{R}}^{(1)}(\chi_s)\right)$	$\ldots$	$\left(\mu_{\mathfrak{R}}^{(m)}(\chi_s), \nu_{\mathfrak{R}}^{(m)}(\chi_s)\right)$

where $0 \leq \mu_{\mathfrak{R}}(\chi) \leq 1$ and $0 \leq \nu_{\mathfrak{R}}(\chi) \leq 1$, depict the membership degree and the non membership degree of $\chi \in \mathfrak{X}$.

In general,

$$\Upsilon_{\mathfrak{R}} = \sqrt[q]{1 - \mu_{\mathfrak{R}}(\chi)^q - \nu_{\mathfrak{R}}(\chi)^q}$$

is said to be hesitancy degree for χ to $\mathfrak{R}$. The pair $(\mu_{\mathfrak{R}}(\chi), \nu_{\mathfrak{R}}(\chi))$, for every $\chi \in \mathfrak{X}$, which is known as q-rung orthopair fuzzy number (q-ROFN). When we concentrate upon the definition of q-ROFS, then we are in a position to split q-ROFS into a class of orthopair fuzzy numbers by assigning different inputs to q. For instance, when $q = 1$ we get intuitionistic fuzzy number (IFN) and if $q = 2$ then we get Pythagorean fuzzy number (PFN). Now we are in a position to say that IFN and PFN both are specific kinds of q-ROFN.

Definition 2. [22] We consider $\mathfrak{X}$ be a classical set and a q-rung orthopair m-polar fuzzy set $\mathfrak{R}$, truncated as (q-ROm-PFS) upon the classical set can be expressed by the mapping $\mu_{\mathfrak{R}}^{(i)} : \mathfrak{X} \to [0,1]$ (known as membership functions) and $\nu_{\mathfrak{R}}^{(i)} : \mathfrak{X} \to [0,1]$ (known as non-membership functions) satisfying the requirements that the aggregate of their q^{th} power will not surpass unity i.e. $\left(\mu_{\mathfrak{R}}^{(i)}(\chi)\right)^q + \left(\nu_{\mathfrak{R}}^{(i)}(\chi)\right)^q \in [0,1]$, ($q > 1$) for $i \in \{1, 2, \ldots, m\}$.

A q-rung orthopair m-polar fuzzy set may be expressed in the form

$$\mathfrak{R} = \left\{ \left\langle \chi, \left(\left(\mu_{\mathfrak{R}}^{(1)}(\chi), \nu_{\mathfrak{R}}^{(1)}(\chi)\right), \ldots, \left(\mu_{\mathfrak{R}}^{(m)}(\chi), \nu_{\mathfrak{R}}^{(m)}(\chi)\right) \right) \right\rangle : \left(\mu_{\mathfrak{R}}^{(i)}(\chi)\right)^q + \left(\nu_{\mathfrak{R}}^{(i)}(\chi)\right)^q \in [0,1], \chi \in \mathfrak{X} \right\}$$

or in more specific format

$$\begin{aligned} \mathfrak{R} &= \left\{ \frac{\chi}{\left(\left(\mu_{\mathfrak{R}}^{(1)}(\chi), \nu_{\mathfrak{R}}^{(1)}(\chi)\right), \ldots, \left(\mu_{\mathfrak{R}}^{(m)}(\chi), \nu_{\mathfrak{R}}^{(m)}(\chi)\right) \right)} : \chi \in \mathfrak{X} \right\} \\ &= \left\{ \frac{\chi}{\left(\left(\mu_{\mathfrak{R}}^{(i)}(\chi), \nu_{\mathfrak{R}}^{(i)}(\chi)\right) \right)} : \chi \in \mathfrak{X}; i \in \{1, 2, \ldots, m\} \right\} \end{aligned}$$

We can express this in another way if $\mathfrak{X} = \{\chi_1, \chi_2, \ldots \chi_s\}$ with the cardinal number s in $\mathfrak{X}$, and its tabular form for $\mathfrak{R}$ is shown as in table 1.

This may also be written in matrix notation as

$$\mathfrak{R} = \begin{bmatrix} \left(\mu_{\mathfrak{R}}^{(1)}(\chi_1), \nu_{\mathfrak{R}}^{(1)}(\chi_1)\right) & \ldots & \left(\mu_{\mathfrak{R}}^{(m)}(\chi_1), \nu_{\mathfrak{R}}^{(m)}(\chi_1)\right) \\ \left(\mu_{\mathfrak{R}}^{(1)}(\chi_2), \nu_{\mathfrak{R}}^{(1)}(\chi_2)\right) & \ldots & \left(\mu_{\mathfrak{R}}^{(m)}(\chi_2), \nu_{\mathfrak{R}}^{(m)}(\chi_2)\right) \\ \vdots & \ddots & \vdots \\ \left(\mu_{\mathfrak{R}}^{(1)}(\chi_s), \nu_{\mathfrak{R}}^{(1)}(\chi_s)\right) & \ldots & \left(\mu_{\mathfrak{R}}^{(m)}(\chi_s), \nu_{\mathfrak{R}}^{(m)}(\chi_s)\right) \end{bmatrix}$$

This $s \times m$ matrix is known as q-ROm-PF-matrix.

5. Correlation coefficient with q-ROm-PFSs

This section is based upon correlation coefficient with q-ROm-PFSs, its functionality and effectiveness in the daily life routine matters. In statistics we discuss the firm relationship between two variable's variation withe the help of correlation coefficient and its value lies in between -1 to +1. A correlation have three kinds.

Positive Correlation:-
In such correlation, both the variables swap alongside and the correlation coefficient lies between 0 and +1. For example, when the hight of a boy increases its weight also increases

Negative Correlation:-
In such correlation, both the variables will contrariwise and the correlation coefficient lies between -1 and 0. For example, the tiredness will be decreases by increasing the ingestion of coffee.

Zero Correlation:-
In such correlation, both the variables are independent. That means there is no relation between them and the correlation coefficient is 0. For example, by consuming more coffee the hight of a boy will not increase.

Definition 3. We consider a q-ROm-PFS $\mathfrak{R}$ upon the classical set χ, where

$$\mathfrak{R} = \left\{ \left\langle \vartheta, \left(\mu^{(i)}(\vartheta), \nu^{(i)}(\vartheta)\right) \right\rangle_{i=1}^{m} : \vartheta \in \chi \right\}.$$

The mean or average of $\mathfrak{R}$ may be discussed as

$$\overline{\mathfrak{R}} = \left\{ \left\langle \left(\overline{\mu^{(i)}}, \overline{\nu^{(i)}}\right) \right\rangle_{i=1}^{m} \right\} \qquad = \left\{ \left\langle \left(\frac{1}{|\chi|} \sum_{\vartheta} \mu^{(i)}(\vartheta), \frac{1}{|\chi|} \sum_{\vartheta} \nu^{(i)}(\vartheta) \right) \right\rangle_{i=1}^{m} \right\}$$

Now we consider $\chi = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5\}$ and q-RO3-PFS as

$$\mathfrak{R} = \begin{pmatrix} \left(\mu^{(1)}(\vartheta_1), \nu^{(1)}(\vartheta_1)\right) & \left(\mu^{(2)}(\vartheta_1), \nu^{(2)}(\vartheta_1)\right) & \left(\mu^{(3)}(\vartheta_1), \nu^{(3)}(\vartheta_1)\right) \\ \left(\mu^{(1)}(\vartheta_2), \nu^{(1)}(\vartheta_2)\right) & \left(\mu^{(2)}(\vartheta_2), \nu^{(2)}(\vartheta_2)\right) & \left(\mu^{(3)}(\vartheta_2), \nu^{(3)}(\vartheta_2)\right) \\ \left(\mu^{(1)}(\vartheta_3), \nu^{(1)}(\vartheta_3)\right) & \left(\mu^{(2)}(\vartheta_3), \nu^{(2)}(\vartheta_3)\right) & \left(\mu^{(3)}(\vartheta_3), \nu^{(3)}(\vartheta_3)\right) \\ \left(\mu^{(1)}(\vartheta_4), \nu^{(1)}(\vartheta_4)\right) & \left(\mu^{(2)}(\vartheta_4), \nu^{(2)}(\vartheta_4)\right) & \left(\mu^{(3)}(\vartheta_4), \nu^{(3)}(\vartheta_4)\right) \\ \left(\mu^{(1)}(\vartheta_5), \nu^{(1)}(\vartheta_5)\right) & \left(\mu^{(2)}(\vartheta_5), \nu^{(2)}(\vartheta_5)\right) & \left(\mu^{(3)}(\vartheta_5), \nu^{(3)}(\vartheta_5)\right) \end{pmatrix}$$

Here, the average of $\mathfrak{R}$ which may be represented as

$$\overline{\mathfrak{R}} = \left(\left(\overline{\mu^{(1)}}, \overline{\nu^{(1)}}\right), \left(\overline{\mu^{(2)}}, \overline{\nu^{(2)}}\right), \left(\overline{\mu^{(3)}}, \overline{\nu^{(3)}}\right) \right)$$

where

$$\begin{aligned} \overline{\mu^{(1)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \mu^{(1)}\vartheta_i), \quad & \overline{\nu^{(1)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \nu^{(1)}(\vartheta_i) \\ \overline{\mu^{(2)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \mu^{(2)}(\vartheta_i), \quad & \overline{\nu^{(2)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \nu^{(2)}(\vartheta_i) \\ \overline{\mu^{(3)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \mu^{(3)}(\vartheta_i), \quad & \overline{\nu^{(3)}} &= \tfrac{1}{5} \textstyle\sum_{i=1}^{5} \nu^{(3)}(\vartheta_i) \end{aligned}$$

Definition 4. We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ, where

$$\mathfrak{R}_A = \begin{pmatrix} \left(\mu_A^{(1)}(\vartheta_1), \nu_A^{(1)}(\vartheta_1)\right) & \cdots & \left(\mu_A^{(m)}(\vartheta_1), \nu_A^{(m)}(\vartheta_1)\right) \\ \left(\mu_A^{(1)}(\vartheta_2), \nu_A^{(1)}(\vartheta_2)\right) & \cdots & \left(\mu_A^{(m)}(\vartheta_2), \nu_A^{(m)}(\vartheta_2)\right) \\ \vdots & \ddots & \vdots \\ \left(\mu_A^{(1)}(\vartheta_j), \nu_A^{(1)}(\vartheta_j)\right) & \cdots & \left(\mu_A^{(m)}(\vartheta_j), \nu_A^{(m)}(\vartheta_j)\right) \end{pmatrix}$$

and

$$\mathfrak{R}_B = \begin{pmatrix} \left(\mu_B^{(1)}(\vartheta_1), \nu_B^{(1)}(\vartheta_1)\right) & \cdots & \left(\mu_B^{(m)}(\vartheta_1), \nu_B^{(m)}(\vartheta_1)\right) \\ \left(\mu_B^{(1)}(\vartheta_2), \nu_B^{(1)}(\vartheta_2)\right) & \cdots & \left(\mu_B^{(m)}(\vartheta_2), \nu_B^{(m)}(\vartheta_2)\right) \\ \vdots & \ddots & \vdots \\ \left(\mu_B^{(1)}(\vartheta_j), \nu_B^{(1)}(\vartheta_j)\right) & \cdots & \left(\mu_B^{(m)}(\vartheta_j), \nu_B^{(m)}(\vartheta_j)\right) \end{pmatrix}.$$

We are defining here the covariance of $\mathfrak{R}_A$ and $\mathfrak{R}_B$ as

$$\mathfrak{C}_{(\mathfrak{R}_A \to \mathfrak{R}_B)} = \frac{1}{3m} \sum_{j=1}^{m} \left(\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} + \mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)} \right)$$

where

$$\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} = \sum_{i=1}^{j} \left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}} \right) \left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}} \right),$$

$$\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)} = \sum_{i=1}^{j} \left(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}} \right) \left(\nu_B^{(j)}(\vartheta_i) - \overline{\nu_B^{(j)}} \right)$$

for $j \in \{1, 2, \ldots, m\}$.
Covariance tells us about the way of linear interrelation betwixt two variables.

Definition 5. We are considering a q-ROm-PFS $\mathfrak{R}_A$ and upon the classical set χ as we have discussed in definition 4 to express the variance as

$$\mathfrak{V}(\mathfrak{R}_A) = \frac{1}{3m} \sum_{j=1}^{m} \left(\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A) + \mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A) \right)$$

where

$$\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A) = \sum_{i=1}^{j} \left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}} \right)^2,$$

$$\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A) = \sum_{i=1}^{j} \left(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}} \right)^2$$

for $j \in \{1, 2, \ldots, m\}$.

Proposition 1. We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ. The following properties hold for variance and covariance for both of these q-ROm-PFSs.

(a) $\mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} \mid \leq \sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}$, $\mid \mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)} \mid \leq \sqrt{\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_B)}$.

(b) $\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_A)} = \mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A),\ \mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_A)} = \mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A)$

(c) $\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} = \mathfrak{V}^{(j)}_{\mu(\mathfrak{R}_B \to \mathfrak{R}_A)}, \mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)} = \mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_B \to \mathfrak{R}_A)}$

Proof. (a) By virtue of famous inequality due to Cauchy-Schwarz, we know that

$$\Big(\sum_{l=1}^{n} c_l d_l\Big)^2 \leq \Big(\sum_{l=1}^{n} c_l^2\Big)\Big(\sum_{l=1}^{n} d_l^2\Big)$$

for $c_l, d_l \in \mathfrak{R}$.
Thus,

$$\begin{aligned}
\Big(\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)}\Big)^2 &= \Big(\sum_{i=1}^{k} \big(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\big)\big(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\big)\Big)^2 \\
&\leq \sum_{i=1}^{k} \big(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\big)^2 \sum_{i=1}^{k} \big(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\big)^2 \\
&= \mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B) \\
\Rightarrow \mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} \mid &\leq \sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A).\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}.
\end{aligned}$$

On the same track we may prove the other part.
The proof of (b) and (c) are quite simple. □

Definition 6. We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ cited in Definition 4. We are defining here the correlation coefficient of $\mathfrak{R}_A$ and $\mathfrak{R}_B$ as

$$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{1}{3m}\sum_{i=1}^{m}\bigg(\varrho_{\mu}^{(i)}(\mathfrak{R}_A, \mathfrak{R}_B) + \varrho_{\nu}^{(i)}(\mathfrak{R}_A, \mathfrak{R}_B)\bigg) \tag{1}$$

where

$$\varrho_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{\sum_{i=1}^{k}\Big(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\Big)\Big(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\Big)}{\sqrt{\sum_{i=1}^{k}\Big(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\Big)^2 \sum_{i=1}^{k}\Big(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\Big)^2}},$$

$$\varrho_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{\sum_{i=1}^{k}\Big(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}}\Big)\Big(\nu_B^{(j)}(\vartheta_i) - \overline{\nu_B^{(j)}}\Big)}{\sqrt{\sum_{i=1}^{k}\Big(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}}\Big)^2 \sum_{i=1}^{k}\Big(\nu_B^{(j)}(\vartheta_i) - \overline{\nu_B^{(j)}}\Big)^2}}$$

and $j \in \{1, 2, \ldots, m\}$.

There are some special cases:

If $\mu_A^{(j)}(\vartheta_i) =$ constant, then $\overline{\mu_A^{(j)}} =$ constant, so that $\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A)} = 0$, and $\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A) = 0$ for every $j \in \{1, 2, \ldots, n\}$. Hence, $\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} = 0$. Thus, we cannot describe the correlation $\varrho_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B)$.

Likewise, if $\nu_A^{(j)}(\vartheta_i) =$ constant for each $i = 1, 2, \ldots, n$, then $\overline{\nu_A^{(j)}} =$ constant and thus $\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A)} = 0$. Hence, $\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A) = 0$ and hence $\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)} = 0$ and $\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A) = 0$. Therefore, we cannot describe the correlation $\varrho_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B)$.

Theorem 1. *We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ, then for any correlation ϱ we have*

1. *ϱ lies among -1 and 1, and may be written as $-1 \leq \varrho(\mathfrak{R}_A, \mathfrak{R}_B) \leq 1$.*

2. *If* $A = B$, *then* $\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = 1$.

3. $\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = 1$ *if* $A = \lambda B$, $\lambda > 0$.

4. ϱ *is symmetric, i.e.* $\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = \varrho(\mathfrak{R}_B, \mathfrak{R}_A)$.

Proof. (1) We have already discussed in the proposition (1) that

$$\begin{aligned}
\mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} \mid &\leq \sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)} \\
\Rightarrow \frac{\mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)} \mid}{\sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}} &\leq 1 \\
\Rightarrow -1 \leq \frac{\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)}}{\sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}} &\leq 1 \\
\Rightarrow -1 \leq \varrho^{(j)}_{\mu}(\mathfrak{R}_A, \mathfrak{R}_B) &\leq 1 \quad .
\end{aligned}$$

Likewise,

$$\varrho^{(j)}_{\nu}(\mathfrak{R}_A, \mathfrak{R}_B) \in [-1, 1].$$

Hence,

$$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) \in [-1, 1].$$

(2) If we consider $A = B$, then $\mu^{(j)}_A(\vartheta_i) = \mu^{(j)}_B(\vartheta_i)$ and $\nu^{(j)}_A(\vartheta_i) = \nu^{(j)}_B(\vartheta_i)$. So,

$$\begin{aligned}
&\varrho^{(j)}_{\mu}(\mathfrak{R}_A, \mathfrak{R}_B) \\
&= \frac{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)}{\sqrt{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)^2 \sum_{i=1}^{k}\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)^2}} \\
&= \frac{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)}{\sqrt{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)^2 \sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)^2}} \\
&= 1.
\end{aligned}$$

Likewise,

$$\varrho^{(j)}_{\nu}(\mathfrak{R}_A, \mathfrak{R}_B) = 1$$

Hence,

$$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

(3) If $A = \lambda B$ then $\mu^{(j)}_A(\vartheta_i) = \lambda\mu^{(j)}_B(\vartheta_i)$, and $\nu^{(j)}_A(\vartheta_i) = \lambda\nu^{(j)}_B(\vartheta_i)$, then

$$\begin{aligned}
&\varrho^{(j)}_{\mu}(\mathfrak{R}_A, \mathfrak{R}_B) \\
&= \frac{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)}{\sqrt{\sum_{i=1}^{k}\left(\mu^{(j)}_A(\vartheta_i) - \overline{\mu^{(j)}_A}\right)^2 \sum_{i=1}^{k}\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)^2}} = \frac{\sum_{i=1}^{k}\left(\lambda\mu^{(j)}_B(\vartheta_i) - \lambda\overline{\mu^{(j)}_B}\right)\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)}{\sqrt{\sum_{i=1}^{k}\left(\lambda\mu^{(j)}_B(\vartheta_i) - \lambda\overline{\mu^{(j)}_B}\right)^2 \sum_{i=1}^{k}\left(\mu^{(j)}_B(\vartheta_i) - \overline{\mu^{(j)}_B}\right)^2}} = 1.
\end{aligned}$$

Likewise,

$$\varrho^{(j)}_{\nu}(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

Hence,

$$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

(4) We can prove it easily. □

Definition 7. We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ. For any correlation coefficient ρ of $\mathfrak{R}_A$ and $\mathfrak{R}_B$ can be designated as

$$\widetilde{\rho}(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{1}{3m} \sum_{j=1}^{m} \left(\widetilde{\rho}_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) + \widetilde{\rho}_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) \right) \tag{2}$$

where

$$\widetilde{\rho}_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{\sum_{i=1}^{k} \left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}} \right) \left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}} \right)}{\max \left\{ \sum_{i=1}^{k} \left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}} \right)^2, \sum_{i=1}^{k} \left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}} \right)^2 \right\}}$$

and

$$\widetilde{\rho}_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{\sum_{i=1}^{k} \left(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}} \right) \left(\nu_B^{(j)}(\vartheta_i) - \overline{\nu_B^{(j)}} \right)}{\max \left\{ \sum_{i=1}^{k} \left(\nu_A^{(j)}(\vartheta_i) - \overline{\nu_A^{(j)}} \right)^2, \sum_{i=1}^{k} \left(\nu_B^{(j)}(\vartheta_i) - \overline{\nu_B^{(j)}} \right)^2 \right\}}.$$

Example 1. Taking q-RO3-PFS over $\chi = \{z_1, z_2, z_3, z_4\}$ as

$$\mathfrak{R}_A = (z_1, 0.337, 0.488), (z_2, 0.225, 0.332), (z_3, 0.722, 0.575)$$

and

$$\mathfrak{R}_B = (z_1, 0.453, 0.514), (z_2, 0.811, 0.110), (z_4, 0.677, 0.266)$$

By using equation (2) we will get the result

$$\mathfrak{C}(\mathfrak{R}_A, \mathfrak{R}_B) = \left[\frac{1}{n} \sum_{z \in \chi} \left(\mu {\mathfrak{R}_A}^q \mu {\mathfrak{R}_A}^q + \nu {\mathfrak{R}_A}^q \nu {\mathfrak{R}_A}^q \right) \right]^{1/q}$$

We can also manage the informational energy for $\mathfrak{R}_A$, i.e.

$$\Gamma(\mathfrak{R}_A) = \mathfrak{C}(\mathfrak{R}_A, \mathfrak{R}_A) = \left[\frac{1}{n} \sum_{z \in \chi} \left(\mu {\mathfrak{R}_A}^{2q} + \nu {\mathfrak{R}_A}^{2q} \right) \right]^{1/q}$$

Also, we have the correlational coefficient as

$$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = \left[\frac{\mathfrak{C}(\mathfrak{R}_A, \mathfrak{R}_B)}{\sqrt{\Gamma(\mathfrak{R}_A)\Gamma(\mathfrak{R}_B)}} \right]$$

So, finding the values

$\mathfrak{C}(\mathfrak{R}_A, \mathfrak{R}_B) = \sqrt[3]{\frac{0.337^3 \times 0.453^3 + \cdots + 0.575^3 \times 0.266^3}{4}} = 0.3844,$

$\Gamma(\mathfrak{R}_A) = \sqrt[3]{\frac{(0.337^6 + 0.488^6 + 0.225^6 + 0.332^6) + (0.722^6 + 0.525^6 + 1)}{4}} = 0.6655,$

$\Gamma(\mathfrak{R}_B) = \sqrt[3]{\frac{(0.453^6 + 0.514^6 + 0.811^6 + 0.110^6) + 1 + (0.677^6 + 0.266^6)}{4}} = 0.7061.$

$\varrho(\mathfrak{R}_A, \mathfrak{R}_B) = \frac{\mathfrak{C}(\mathfrak{R}_A, \mathfrak{R}_B)}{\sqrt{\Gamma(\mathfrak{R}_A)\Gamma(\mathfrak{R}_B)}} = 0.5608.$

Definition 8. Let $\mathfrak{R}_A$ and $\mathfrak{R}_B$ be two q-ROm-PFSs as cited in Definition 4. We choose the weights referred as "ω_i"for $i \in \{1, 2, \ldots, n\}$ having the obvious restrictions that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0, 1]$. We are also well aware that ϑ_i for $i = \in \{1, 2, \ldots, n\}$ as alternatives are also affected by these given weights ω_i.

Now we are in a position to discuss the weighted covariance of $\mathfrak{R}_A$ and $\mathfrak{R}_B$ as

$${}^{\omega}\mathfrak{C}_{(\mathfrak{R}_A \to \mathfrak{R}_B)} = \frac{1}{3m} \sum_{j=1}^{m} \left({}^{\omega}\mathfrak{C}_{\mu(\mathfrak{R}_A \to \mathfrak{R}_B)}^{(j)} + {}^{\omega}\mathfrak{C}_{\nu(\mathfrak{R}_A \to \mathfrak{R}_B)}^{(j)} \right) \tag{3}$$

We have here,

$$ {}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\to\mathfrak{R}_B)} = \sum_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right), $$

$$ {}^{\omega}\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A\to\mathfrak{R}_B)} = \sum_{i=1}^{k}\omega_i\left(\nu_A^{(j)}(\vartheta_i)-\overline{\nu_A^{(j)}}\right)\left(\nu_B^{(j)}(\vartheta_i)-\overline{\nu_B^{(j)}}\right) $$

for every $j \in \{1,2,\ldots,m\}$.

Definition 9. We are considering a q-ROm-PFS $\mathfrak{R}_A$ upon the classical set χ. We are suggesting here the weights referred as "ω_i"for $i \in \{1,2\ldots,n\}$ having the restrictions that $\sum_{i=1}^{n}\omega_i = 1$ and each $\omega_i \in [0,1]$. We are also well aware that selection of "ϑ_i"for $i \in \{1,2\ldots,n\}$ as alternative is also affected by these given weights ω_i.
Now we are in a position to discuss the weighted variance of $\mathfrak{R}_A$.

$$ {}^{\omega}\mathfrak{V}(\mathfrak{R}_A) = \frac{1}{3m}\sum_{j=1}^{m}\left({}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A) + {}^{\omega}\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A)\right) $$

We have here,

$$ {}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A) = \sum_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)^2, $$

$$ {}^{\omega}\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A) = \sum_{i=1}^{k}\omega_i\left(\nu_A^{(j)}(\vartheta_i)-\overline{\nu_A^{(j)}}\right)^2 $$

for every $j \in \{1,2,\ldots,m\}$.

Theorem 2. *We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ, then the correlation $\widetilde{\rho}$ have these results:*

(a) $\widetilde{\rho}$ *is symmetric, i.e.* $\widetilde{\rho}(\mathfrak{R}_A,\mathfrak{R}_B) = \widetilde{\rho}(\mathfrak{R}_B,\mathfrak{R}_A)$.

(b) $\widetilde{\rho}$ *must be in* $[-1,1]$, *i.e.* $-1 \le \widetilde{\rho}(\mathfrak{R}_A,\mathfrak{R}_B) \le 1$.

(c) $\widetilde{\rho}(\mathfrak{R}_A,\mathfrak{R}_B) = 1$ *when* $A = B$.

(d) $\widetilde{\rho}(\mathfrak{R}_A,\mathfrak{R}_B) = 1$ *when* $A = \lambda B$, *for* $\lambda > 0$.

Proof. (a) may be proved easily.
(b) As we have already discussed in proposition **??** that

$$ \mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\to\mathfrak{R}_B)} \mid \le \sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)} $$

$$ \Rightarrow \frac{\mid \mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\to\mathfrak{R}_B)} \mid}{\sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}} \le 1 $$

$$ \Rightarrow -1 \le \frac{\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\to\mathfrak{R}_B)}}{\sqrt{\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}} \le 1 $$

$$ \Rightarrow -1 \le \widetilde{\rho}^{(j)}_{\mu}(\mathfrak{R}_A,\mathfrak{R}_B) \le 1. $$

Likewise,

$$-1 \leq \widetilde{\rho}_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) \leq 1.$$

Thus, we have

$$-1 \leq \widetilde{\rho}(\mathfrak{R}_A, \mathfrak{R}_B) \leq 1.$$

(c) When we have $A = B$, then $\mu_A^{(j)}(\vartheta_i) = \mu_B^{(j)}(\vartheta_i)$, and $\nu_A^{(j)}(\vartheta_i) = \nu_B^{(j)}(\vartheta_i)$, so that

$$\begin{aligned}
&\widetilde{\vartheta}_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) \\
&= \frac{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)}{\max\left\{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)^2, \sum_{i=1}^{k}\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)^2\right\}} \\
&= \frac{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)}{\max\left\{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)^2, \sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)^2\right\}} \qquad = 1.
\end{aligned}$$

Likewise,

$$\widetilde{\rho}_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

Therefore, we have

$$\widetilde{\rho}(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

(d) When we take $A = \lambda B$, then we have $\mu_A^{(j)}(\vartheta_i) = (\lambda)\mu_B^{(j)}(\vartheta_i)$, $\nu_A^{(j)}(\vartheta_i) = (\lambda)\nu_B^{(j)}(\vartheta_i)$, so that

$$\begin{aligned}
&\widetilde{\rho}_{\mu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) \\
&= \frac{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)}{\max\left\{\sum_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i) - \overline{\mu_A^{(j)}}\right)^2, \sum_{i=1}^{k}\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)^2\right\}} \\
&= \frac{\sum_{i=1}^{k}\left((\lambda)\mu_B^{(j)}(\vartheta_i) - (\lambda)\overline{\mu_B^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)}{\max\left\{\sum_{i=1}^{k}\left((\lambda)\mu_B^{(j)}(\vartheta_i) - (\lambda)\overline{\mu_B^{(j)}}\right)^2, \sum_{i=1}^{k}\left(\mu_B^{(j)}(\vartheta_i) - \overline{\mu_B^{(j)}}\right)^2\right\}} \\
&= 1.
\end{aligned}$$

Likewise,

$$\widetilde{\rho}_{\nu}^{(j)}(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

Thus, we have

$$\widetilde{\rho}(\mathfrak{R}_A, \mathfrak{R}_B) = 1.$$

□

The concept of uncertainty performs a significant part in decision making. The weights also produce effects in the decision making. The mathematical models require data which is established properly. To overcome such problems, here we are introducing Weighted Correlation Coefficients (WCCs).

Proposition 2. *We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ upon the classical set χ. The following properties hold for weighted variance and weighted covariance for these both q-ROm-PFSs.*

(a) $\mid {}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\rightarrow\mathfrak{R}_B)} \mid\leq \sqrt{{}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\,{}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}$, $\mid {}^{\omega}\mathfrak{C}^{(j)}_{\nu({}^{\omega}\mathfrak{R}_A\rightarrow\mathfrak{R}_B)} \mid\leq \sqrt{{}^{\omega}\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A)\,{}^{\omega}\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_B)}$.

(b) ${}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\rightarrow\mathfrak{R}_A)} = {}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)$, ${}^{\omega}\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A\rightarrow\mathfrak{R}_A)} = {}^{\omega}\mathfrak{V}^{(j)}_{\nu}(\mathfrak{R}_A)$

(c) ${}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\rightarrow\mathfrak{R}_B)} = {}^{\omega}\mathfrak{V}^{(j)}_{\mu(\mathfrak{R}_B\rightarrow\mathfrak{R}_A)}$,
${}^{\omega}\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_A\rightarrow\mathfrak{R}_B)} = {}^{\omega}\mathfrak{C}^{(j)}_{\nu(\mathfrak{R}_B\rightarrow\mathfrak{R}_A)}$.

Proof. (a) As we have already discussed Cauchy-Schwarz inequality, i.e.

$$\Big(\sum_{l=1}^{n} c_l d_l\Big)^2 \leq \Big(\sum_{l=1}^{n} c_l^2\Big)\Big(\sum_{l=1}^{n} d_l^2\Big)$$

for $c_l, d_l \in \mathfrak{R}$. We are using this Cauchy-Schwarz inequality to verify our result (a).
Consider

$$\begin{aligned}
&\Big({}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\rightarrow\mathfrak{R}_B)}\Big)^2 \\
&= \Big(\sum_{i=1}^{k}\omega_i\big(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\big)\big(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\big)\Big)^2 \\
&= \Big(\sum_{i=1}^{k}\sqrt{\omega_i}\big(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\big)\sqrt{\omega_i}\big(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\big)\Big)^2 \\
&\leq \sum_{i=1}^{k}\omega_i\big(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\big)^2\sum_{i=1}^{k}\omega_i\big(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\big)^2 \\
&= {}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\,{}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B) \\
&\Rightarrow\mid {}^{\omega}\mathfrak{C}^{(j)}_{\mu(\mathfrak{R}_A\rightarrow\mathfrak{R}_B)} \mid\leq\sqrt{{}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_A)\,{}^{\omega}\mathfrak{V}^{(j)}_{\mu}(\mathfrak{R}_B)}
\end{aligned}$$

On the same track we may prove the other part.
(b) and (c) can be proved in simple way. □

Definition 10. We are considering two q-ROm-PFSs $\mathfrak{R}_A$ and $\mathfrak{R}_B$ given in Definition 4. We express the weighted correlation coefficient(WCC)of $\mathfrak{R}_A$ and $\mathfrak{R}_B$ as

$$^{\omega}\varrho(\mathfrak{R}_A,\mathfrak{R}_B) = \frac{1}{3m}\sum_{j=1}^{m}\Big({}^{\omega}\varrho^{(j)}_{\mu}(\mathfrak{R}_A,\mathfrak{R}_B) + {}^{\omega}\varrho^{(j)}_{\nu}(\mathfrak{R}_A,\mathfrak{R}_B)\Big) \tag{4}$$

with

$$
\begin{aligned}
&{}^{\omega}\varrho_{\mu}^{(j)}(\mathfrak{R}_A,\mathfrak{R}_B)\\
&=\frac{\sum\limits_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)}{\sqrt{\sum\limits_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)^2\sum\limits_{i=1}^{k}\omega_i\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)^2}},\\
&{}^{\omega}\varrho_{\nu}^{(j)}(\mathfrak{R}_A,\mathfrak{R}_B)\\
&=\frac{\sum\limits_{i=1}^{k}\omega_i\left(\nu_A^{(j)}(\vartheta_i)-\overline{\nu_A^{(j)}}\right)\left(\nu_B^{(j)}(\vartheta_i)-\overline{\nu_B^{(j)}}\right)}{\sqrt{\sum\limits_{i=1}^{k}\omega_i\left(\nu_A^{(j)}(\vartheta_i)-\overline{\nu_A^{(j)}}\right)^2\sum\limits_{i=1}^{k}\omega_i\left(\nu_B^{(j)}(\vartheta_i)-\overline{\nu_B^{(j)}}\right)^2}}
\end{aligned}
$$

for every $j \in \{1,2,\ldots,m\}$.

Theorem 3. *Let $\mathfrak{R}_A$ and $\mathfrak{R}_B$ be as given in Definition 4. Then,*

(a) Weighted correlation ${}^{\omega}\varrho$ always falls in between -1 and 1, i.e. $-1 \leq {}^{\omega}\varrho(\mathfrak{R}_A,\mathfrak{R}_B) \leq 1$.

(b) ${}^{\omega}\varrho(\mathfrak{R}_A,\mathfrak{R}_B) = 1$ when $A = B$.

(c) ${}^{\omega}\varrho(\mathfrak{R}_A,\mathfrak{R}_B) = 1$ when $A = \lambda B$, $\lambda > 0$.

Proof. The proof follows a similar structure to the proof of proposition 4. □

The role of weights in correlation is undeviating. To show this situation we observe that the weighted alternatives can be observed as dissimilar.

$$
\begin{aligned}
&{}^{\omega}\varrho_{\mu}^{(j)}(\mathfrak{R}_A,\mathfrak{R}_B)\\
&=\frac{\sum\limits_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)}{\sqrt{\sum_{i=1}^{k}\omega_i\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)^2\sum\limits_{i=1}^{k}\omega_i\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)^2}}\\
&=\frac{\sum\limits_{i=1}^{k}\frac{1}{n}\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)}{\sqrt{\sum\limits_{i=1}^{k}\frac{1}{n}\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)^2\sum\limits_{i=1}^{k}\frac{1}{n}\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)^2}}\\
&=\frac{\sum\limits_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)}{\sqrt{\sum\limits_{i=1}^{k}\left(\mu_A^{(j)}(\vartheta_i)-\overline{\mu_A^{(j)}}\right)^2\sum\limits_{i=1}^{k}\left(\mu_B^{(j)}(\vartheta_i)-\overline{\mu_B^{(j)}}\right)^2}}\\
&=\rho_{\mu}^{(j)}(\mathfrak{R}_A,\mathfrak{R}_B).
\end{aligned}
$$

6. Practical Utility of the Proposed Correlation Measures

Correlation is actually a statistics based topic and it plays a dynamic role in innumerable fields of our routine life. Correlation may be applied in psychology, engineering and in so many fields of medical sciences. Correlation supports us to collect required information about the topic. These information may be materialized without any experimental work. Correlation measure can be used ideally for collecting data in an ordinary way. We can use this data collection to generalize our findings to solve the real life related issues. Correlation may be used in multi criteria decision making (MCDM) issues. The most essential thing is the data collection that will provide the validity and reliability about the results.

At present, we are going to use correlation in pattern recognition and a usage in medical diagnosis. Pattern recognition is usually used to analyze the data. Algorithms based upon pattern recognition furnish the appropriate result for all provided inputs to carry out "most likely" to contest the given intuition considering to contemplate the data. A customary illustration for pattern recognition is known as "deep learning "that delivers corroboration to obtain the massive and dependable outcomes.

6.1 Execution for Pattern Recognition

Before we start the execution of pattern recognition, we establish an Algorithm for this purpose that will support us in a way to apply q-ROm-PFSs in our proposed decision making and receiving the required information.

Algorithm: (To recognize the Pattern)

Step 1: Study the case carefully to recognize the pattern as a q-ROm-PFS for $\mathfrak{B}$.

Step 2: We have to manage a well known pattern in a sequence format as $\mathfrak{B}^{(1)}, \mathfrak{B}^{(2)}, \cdots, \mathfrak{B}^{(n)}$.

Step 3: As the format already discussed $\mathfrak{B}^{(1)}, \mathfrak{B}^{(2)}, \cdots, \mathfrak{B}^{(n)}$ may be used to discover the correlation coefficient with the help of formula either (1) or (2).

Step 4: If we consider a q-ROm-PFS upon the classical set χ have weighted alternatives, then the weighted correlation coefficient formulas either (3) or (4) may be used.

Step 5: A recognized pattern $\mathfrak{B}$ related to $\mathfrak{B}^{(j)}$, where $\mathfrak{B}^{(j)}$ provides the largest correlation with respect to $\mathfrak{B}$.

Example 2. We analyze pattern recognition in this example. It is a matter of great concern that how we use pattern recognition in our daily routine life. Pattern recognition is a way to recognize a number of patterns by the use of machine learning algorithms. Pattern recognition may be used in speaker identification, voice recognition, machine aided medical diagnosis and multimedia document recognition. We collect data that is known as raw data may be transferred to machine language. It also includes cluster and classification of patterns. An utmost vital and substantial task in machine learning, robotic and security management. We consider a particular issue of pattern recognition, also known as, facial recognition.

To identify a recognition method we are discussing different options for the use of fingerprints. Fingerprint is a multifaceted and surprisingly supple method for personage identification. It's an alternate way of key cards or passwords. Nowadays, the most advanced system of fingerprint is used in smart phones. As the fingerprints of every person in the world are unique, so this way of identification may be used exceptionally in every field of life for the recognition of human beings. Due to its practicality, we are free to move in the markets without carrying money, wallets and credit/debit cards.

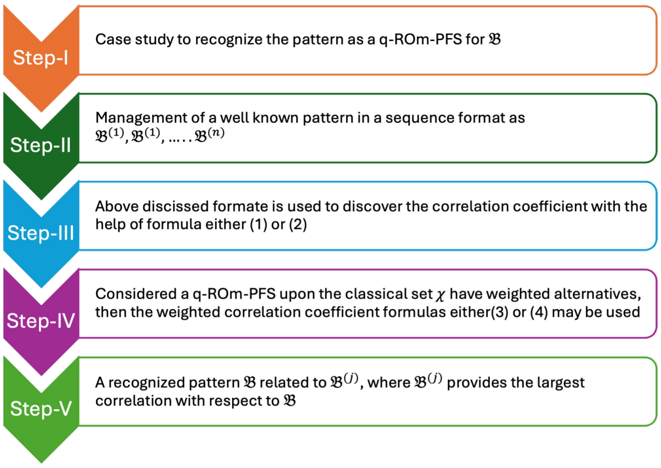

Fig. 1. Flow chart of Algorithm: To recognize the Pattern.

This process is based upon pattern recognition. This recognizing technology is most authoritative in the field of biometric applications. The use of biometric device provide us timeliness and swiftness, wherever the purchasers feel a sense of modernity when buying some items with the move of a finger.

We are considering a biometric device, where we have inserted three different patterns $\mathfrak{B}^{(1)}$, $\mathfrak{B}^{(2)}$ and $\mathfrak{B}^{(3)}$ in the form of q-ROm-PFS for three distinct individuals,to recognize a person. The fingerprint will be examined by the device, first to scan the finger then scrutinize about the unknown person, peruse and transform the data into a q-ROm-PFS model, lets pick $\mathfrak{B}$, that may be presented as in Table 2.

Table 2
Standard q-ROm-PFS $\mathfrak{B}$

$\mathfrak{B}$	q-RO3-PFS
ϑ_1	$(0.222, 0.306), (0.306, 0.599), (0.703, 0.015)$
ϑ_2	$(0.431, 0.366), (0.900, 0.088), (0.763, 0.222)$
ϑ_3	$(0.402, 0.500), (0.112, 0.464), (0.811, 0.135)$

Table 3
Appraisal for pattern $\mathfrak{B}^{(1)}$ in the form of q-ROm-PFS

$\mathfrak{B}^{(1)}$	q-RO3-PFS
ϑ_1	$(0.506, 0.421), (0.480, 0.352), (0.600, 0.234)$
ϑ_2	$(0.700, 0.111), (0.566, 0.222), (0.498, 0.378)$
ϑ_3	$(0.333, 0.444), (0.311, 0.600), (0.118, 0.559)$

To establish a mathematical model, we take three different patterns as shown in Table 3, Table 4 and Table 5.

Here we are in a position to sort out the correlations of an unrevealed pattern by considering a well known pattern, to find out that which pattern is well matched to the unfamiliar pattern. By using the algorithm we have to feed the whole procedure in the machine to find out the related personal.

The correlation coefficients for the recognition $\mathfrak{B}$ for $\mathfrak{B}^{(1)}, \mathfrak{B}^{(2)}, \mathfrak{B}^{(3)}$ (by applying these formula (1) and (2) respectively) represented as in Table 6.

But when the alternatives $\vartheta_1, \vartheta_2, \vartheta_3$ having weights $\omega = (0.24, 0.36, 0.40)$, at that time the weighted correlation coefficients may be calculated (using the formula (3) and (4) respectively) as in Table 7.

Table 4
Appraisal for pattern $\mathfrak{B}^{(2)}$ in the form of q-ROm-PFS

$\mathfrak{B}^{(2)}$	q-RO3-PFS
ϑ_1	$(0.511, 0.114), (0.123, 0.744), (0.421, 0.456)$
ϑ_2	$(0.087, 0.799), (0.436, 0.047), (0.330, 0.049)$
ϑ_3	$(0.811, 0.224), (0.515, 0.100), (0.802, 0.122)$

Table 5
Appraisal for pattern $\mathfrak{B}^{(3)}$ in the form of q-ROm-PFS

$\mathfrak{B}^{(3)}$	q-RO3-PFS
ϑ_1	$(0.418, 0.133), (0.542, 0.116), (0.703, 0.204)$
ϑ_2	$(0.332, 0.235), (0.403, 0.206), (0.434, 0.015)$
ϑ_3	$(0.700, 0.113), (0.453, 0.015), (0.328, 0.048)$

The correlation coefficients of $\mathfrak{B}$ from the known patterns $\mathfrak{B}^{(1)}$, $\mathfrak{B}^{(2)}$ and $\mathfrak{B}^{(3)}$ under the proposed correlations are given in the Table 8.

The results presented in Table 8 collectively indicate that the pattern $\mathfrak{B}$ finds the largest correlation from $\mathfrak{B}^{(3)}$. Hence, the facial recognition machine spots the person $\mathfrak{B}$ as $\mathfrak{B}^{(3)}$, and then the evaluators determines the physical presence of the anonymous individual.

Also to demonstrate the practical utility of the proposed correlation measures for q-rung orthopair m-polar fuzzy sets (q-ROm-PFSs), we consider an application in the domain of pattern recognition.

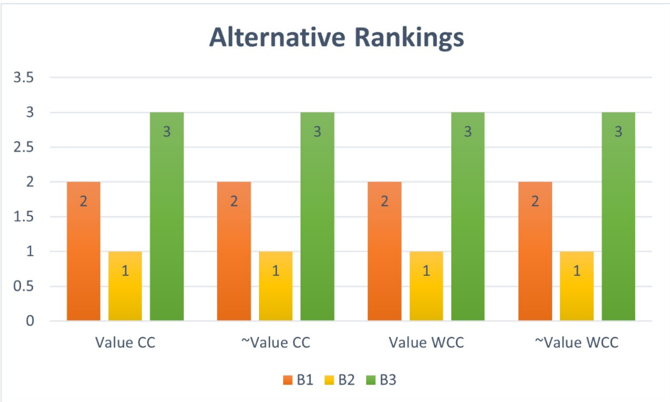

Fig. 2. Alternative ranking for the different values.

Pattern recognition is a crucial technique in various fields, such as biometrics, computer vision, and machine learning, where the goal is to identify and categorize objects, signals, or behaviors based on observed data. In this example, we will focus on the problem of facial recognition, which involves matching an unknown facial pattern to a set of known patterns stored in a database.

Let's assume we have a biometric device that has been programmed to store three different facial patterns, denoted as $B^{(1)}$, $B^{(2)}$, and $B^{(3)}$, represented in the form of q-ROm-PFSs. These patterns correspond to three distinct individuals, as shown in Tables 3, 4, and 5, respectively.

Now, the device is presented with an unknown facial pattern, denoted as B, which needs to be recognized and matched to one of the stored patterns. The q-ROm-PFS representation of the unknown pattern B is given in Table 2.

Table 6
Values of correlation coefficients betwixt $\mathfrak{B}$ and $\mathfrak{B}^k$, $k = 1, 2, 3$

$\varrho(\mathfrak{B}, \mathfrak{B}^{(1)}) = 0.09$	$\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(1)}) = 0.07$
$\varrho(\mathfrak{B}, \mathfrak{B}^{(2)}) = -0.29$	$\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(2)}) = -0.28$
$\varrho(\mathfrak{B}, \mathfrak{B}^{(3)}) = \mathbf{0.36}$	$\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(3)}) = \mathbf{0.23}$

Table 7
Values of weighted correlation coefficients betwixt $\mathfrak{B}$ and $\mathfrak{B}^k$, $k = 1, 2, 3$

${}^{\omega}\varrho(\mathfrak{B}, \mathfrak{B}^{(1)}) = 0.10$	${}^{\omega}\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(1)}) = 0.06$
${}^{\omega}\varrho(\mathfrak{B}, \mathfrak{B}^{(2)}) = -0.31$	${}^{\omega}\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(2)}) = -0.29$
${}^{\omega}\varrho(\mathfrak{B}, \mathfrak{B}^{(3)}) = \mathbf{0.39}$	${}^{\omega}\widetilde{\rho}(\mathfrak{B}, \mathfrak{B}^{(3)}) = \mathbf{0.27}$

To determine the best match, we will calculate the correlation coefficients between the unknown pattern B and each of the known patterns $B^{(1)}$, $B^{(2)}$, and $B^{(3)}$, using the proposed correlation measures (Equations 1 and 2).

The unweighted correlation coefficients between B and the known patterns are presented in Table 6. The results show that pattern B has the highest correlation with $B^{(3)}$, indicating that the unknown facial pattern is most similar to the third stored pattern.

Additionally, we can consider the case where the alternatives (facial features) have different weights, reflecting their relative importance in the recognition process and shown in the figure 2. In such a scenario, we can employ the weighted correlation measure (Equation 3), as shown in Table 7, assuming a weight vector $\omega = (0.24, 0.36, 0.40)$.

The results in Table 8 confirm that the unknown pattern B has the strongest correlation with the third stored pattern $B^{(3)}$, under both the unweighted and weighted correlation measures. Therefore, the facial recognition device will identify the unknown individual as the person corresponding to the third stored pattern.

This application example demonstrates the effectiveness of the proposed correlation measures in the context of pattern recognition, where the q-ROm-PFS framework allows for the representation of complex, multi-polar facial features, and the correlation analysis enables the identification of the closest matching pattern and its comparison shown in the figure 3. The ability to incorporate weighted components further enhances the decision-making capabilities of the recognition system.

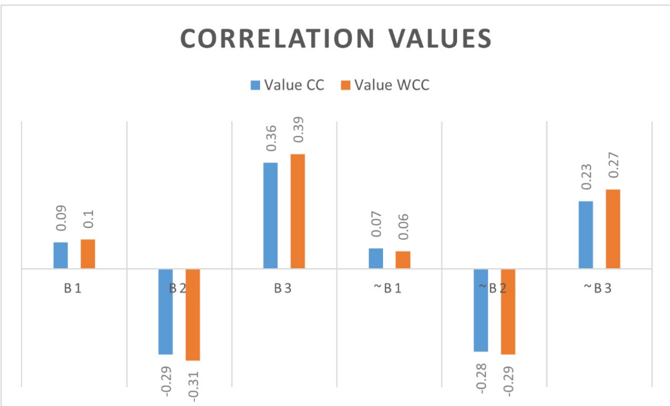

Fig. 3. Comparison for the correlation values.

The comprehensive case study presented here illustrates the practical utility of the developed correlation measures and their potential to be applied in various real-world pattern recognition problems involving uncertain, imprecise, and multi-faceted data.

Table 8
Comparison analysis

Correlation measures	Ranking	Optimal pattern
ρ	$\mathfrak{B}^{(3)} \succ \mathfrak{B}^{(1)} \succ \mathfrak{B}^{(2)}$	$\mathfrak{B}^{(3)}$
$\widetilde{\varrho}$	$\mathfrak{B}^{(3)} \succ \mathfrak{B}^{(1)} \succ \mathfrak{B}^{(2)}$	$\mathfrak{B}^{(3)}$
${}^{\omega}\rho$	$\mathfrak{B}^{(3)} \succ \mathfrak{B}^{(1)} \succ \mathfrak{B}^{(2)}$	$\mathfrak{B}^{(3)}$
${}^{\omega}\widetilde{\varrho}$	$\mathfrak{B}^{(3)} \succ \mathfrak{B}^{(1)} \succ \mathfrak{B}^{(2)}$	$\mathfrak{B}^{(3)}$

7. Conclusion

MCDM methodology have been explored by various researchers. The methodologies outlined to achieve this end are usually affected by the judgment operational structure employed. Majority of its main problems are under the influence of imprecise, vague, uncertain, and multi-polar statistics which cannot be elaborated to an acceptable degree through fuzzy set only. An m-polar fuzzy set (m-PFS) has the capability to handle imprecision by multi-polarity accompanied by the framework of q-rung orthopair fuzzy environment. To cope with realistic uncertain circumstances, we make use of q-rung orthopair m-polar fuzzy environment. We offer two novel correlation measures (unweighted and weighted) in this context. Certain characteristics of covariances and correlation measures are also studied. The major mastery of the suggested correlation measures is that these can be utilized dexterously in the presence of uncertain multipolar information. We posited practical utilization of the suggested techniques in pattern identification.

The key innovations and contributions of the current research work lie in the development of novel correlation and weighted correlation measures specifically tailored to the q-rung orthopair m-polar fuzzy set (q-ROm-PFS) framework. This represents a significant advancement compared to the existing literature on correlation measures in fuzzy and multi-polar fuzzy settings.

The primary innovation of this research is the extension of correlation analysis to the generalized q-ROm-PFS construct. Previous studies have explored correlation measures in the context of classical fuzzy sets, intuitionistic fuzzy sets (IFSs), and Pythagorean fuzzy sets (PFSs). However, the q-ROm-PFS framework offers a more robust and flexible representation of uncertainty, as it combines the capabilities of q-rung orthopair fuzzy sets and m-polar fuzzy sets.

By proposing correlation measures for q-ROm-PFSs, the current work overcomes the limitations of existing approaches, which were primarily designed for less generalized fuzzy set extensions. The q-ROm-PFS framework allows for a broader range of membership and non-membership degree combinations, as well as the incorporation of multi-polarity, which is particularly important in real-world decision-making and pattern recognition problems.

The unique theoretical properties and behavioral characteristics of the proposed correlation and weighted correlation measures are another key innovation. These measures satisfy desirable mathematical properties, such as symmetry, boundedness, and normalization, ensuring their theoretical validity and applicability. Moreover, the weighted correlation measure introduces the ability to incorporate the relative importance of different components, which is crucial in multi-criteria decision-making scenarios.

Another significant contribution of this research is the demonstration of the practical utility of the proposed correlation measures in pattern recognition problems. The comprehensive case study on facial recognition, involving the identification of an unknown pattern among a set of known patterns, showcases the effectiveness of the q-ROm-PFS-based correlation analysis. This application highlights the potential for the developed measures to enable new types of analysis, modeling, and insights that were not feasible with previous correlation measures designed for less generalized fuzzy set frameworks.

Furthermore, the comparative analysis presented in the paper, which compares the results obtained using the proposed measures against other well-established MCDM techniques, such as TOPSIS and VIKOR, further underscores the unique strengths and advantages of the q-ROm-PFS-based correlation measures. This comparative assessment helps position the current work within the broader context of decision-making under uncertainty. In summary, the key innovations and contributions of this research include:

- The extension of correlation analysis to the q-rung orthopair m-polar fuzzy set (q-ROm-PFS) framework, which offers a more comprehensive and flexible representation of uncertainty compared to previous fuzzy set extensions.
- The development of novel correlation and weighted correlation measures for q-ROm-PFSs, which satisfy desirable mathematical properties and overcome the limitations of existing measures.
- The incorporation of the multi-polarity aspect into the correlation analysis, enabling the handling of complex, multi-faceted information in decision-making and pattern recognition problems.
- The demonstration of the practical utility of the proposed measures in a comprehensive case study on facial recognition, showcasing their effectiveness and potential for enabling new types of analysis and insights.
- The comparative analysis against other MCDM techniques, which highlights the unique strengths and advantages of the q-ROm-PFS-based correlation measures.

These innovations and contributions significantly advance the state of the art in correlation analysis within the realm of generalized fuzzy set theory, with potential applications across various domains, including decision-making, pattern recognition, and beyond.

Conflict of interest
The authors declare no conflicts of interest.

References

[1] Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1(1), 3-28.

[2] Feng, F., Zheng, Y., Sun, B., & Akram, M. (2022). Novel score functions of generalized orthopair fuzzy membership grades with application to multiple attribute decision making. Granular Computing, 7(1), 95-111.

[3] Akram, M., Ali, G., & Shabir, M. (2021). A hybrid decision-making framework using rough m F bipolar soft environment. Granular Computing, 6, 539-555.

[4] Arya, V., & Kumar, S. (2021). Knowledge measure and entropy: a complementary concept in fuzzy theory. Granular Computing, 6(3), 631-643.

[5] Atanassov, K. T. (1999). Intuitionistic fuzzy sets (pp. 1-137). Physica-Verlag HD.

[6] Atanassov K. T., & Stoeva, S. (1983). Intuitionistic fuzzy sets. In Polish Symposium on Interval & Fuzzy Mathematics, Poznan.

[7] Yager, R. R. (2013). Pythagorean fuzzy subsets. In 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS) (pp. 57-61). IEEE.

[8] Farhadinia, B., Effati, S., & Chiclana, F. (2021). A family of similarity measures for q-rung orthopair fuzzy sets and their applications to multiple criteria decision making. International Journal of Intelligent Systems, 36(4), 1535-1559.

[9] Peng, X., Yuan, H., & Yang, Y. (2017). Pythagorean fuzzy information measures and their applications. International Journal of Intelligent Systems, 32(10), 991-1029.

[10] Garg, H., & Chen, S. M. (2020). Multiattribute group decision making based on neutrality aggregation operators of q-rung orthopair fuzzy sets. Information Sciences, 517, 427-447.

[11] Gupta, R., & Kumar, S. (2022). A new similarity measure between picture fuzzy sets with applications to pattern recognition and clustering problems. Granular Computing, 7(3), 561-576.

[12] Kadian, R., & Kumar, S. (2022). New fuzzy mean codeword length and similarity measure. Granular Computing, 7(3), 461-478.

[13] Liu, S., Wang, B., & Liu, J. (2020). Group decision making under social influences based on information entropy. Granular Computing, 5, 303-308.

[14] Naeem, K., Riaz, M., & Afzal, D. (2019). Pythagorean m-polar fuzzy sets and TOPSIS method for the selection of advertisement mode. Journal of Intelligent & Fuzzy Systems, 37(6), 8441-8458.

[15] Zhang, S., Hou, Y., Zhang, S., & Zhang, M. (2017). Fuzzy control model and simulation for nonlinear supply chain system with lead times. Complexity, 2017(1), 2017634.

[16] Haq, H. B. U., Akram, W., Irshad, M. N., Kosar, A., & Abid, M. (2024). Enhanced real-time facial expression recognition using deep learning. Acadlore Transactions on AI and Machine Learning, 3(1), 24-35.

[17] Naeem, K., & Divvaz, B. (2023). Information measures for MADM under m-polar neutrosophic environment. Granular Computing, 8(3), 597-616.

[18] Kumam, W., Naeem, K., Riaz, M., Khan, M. J., & Kumam, P. (2023). Comparison measures for Pythagorean m-polar fuzzy sets and their applications to robotics and movie recommender system. Applied Mathematics for Modern Challenges, 8(5).

[19] Yager, R. R. (2016). Generalized orthopair fuzzy sets. IEEE transactions on fuzzy systems, 25(5), 1222-1230.

[20] Rani, P., Jain, D., & Hooda, D. S. (2019). Extension of intuitionistic fuzzy TODIM technique for multi-criteria decision making method based on shapley weighted divergence measure. Granular Computing, 4, 407-420.

[21] Riaz, M., Naeem, K., & Afzal, D. (2020). A similarity measure under pythagorean fuzzy soft environment with applications. Computational and Applied Mathematics, 39(4), 269.

[22] Singh, S., & Ganie, A. H. (2020). On some correlation coefficients in Pythagorean fuzzy environment with applications. International Journal of Intelligent Systems, 35(4), 682-717.

[23] Singh, S., & Ganie, A. H. (2022). Generalized hesitant fuzzy knowledge measure with its application to multi-criteria decision-making. Granular Computing, 7(2), 239-252.

[24] Riaz, M., Garg, H., Hamid, M. T., & Afzal, D. (2022). Modelling uncertainties with TOPSIS and GRA based on q-rung orthopair m-polar fuzzy soft information in COVID-19. Expert Systems, 39(5), e12940.

[25] Abid, M., & Shahid, M. (2024). Tumor Detection in MRI Data using Deep Learning Techniques for Image Classification and Semantic Segmentation. Sustainable Machine Intelligence Journal, 9, 1-13.

[26] Hamid, M. T., Riaz, M., & Naeem, K. (2022). q-Rung orthopair fuzzy soft topology with multi-attribute decision-making. In q-Rung Orthopair Fuzzy Sets: Theory and Applications (pp. 17-46). Singapore: Springer Nature Singapore.

[27] Riaz, M., Hamid, M. T., Afzal, D., Pamucar, D., & Chu, Y. M. (2021). Multi-criteria decision making in robotic agri-farming with q-rung orthopair m-polar fuzzy sets. PLoS one, 16(2), e0246485.

[28] Hamid, M. T., Riaz, M., & Naeem, K. (2022). A study on weighted aggregation operators for q-rung orthopair m-polar fuzzy set with utility to multistage decision analysis. International Journal of Intelligent Systems, 37(9), 6354-6387.

[29] Mitchell, H. (2004). A correlation coefficient for intuitionistic fuzzy sets. International journal of intelligent systems, 19(5), 483-490.

[30] Meharunnisa, M. S., Abid, M., Awais, M., & Stevic, Z. (2023). Analysis of software effort estimation by machine learning techniques. Ingénierie des Systèmes d'Information, 28(6), 1445-1457.

[31] Zhang, S., Zhang, P., & Zhang, M. (2019). Fuzzy emergency model and robust emergency strategy of supply chain system under random supply disruptions. Complexity, 2019(1), 3092514.

[32] Ejegwa, P. A., Onyeke, I. C., & Adah, V. (2020). An algorithm for an improved intuitionistic fuzzy correlation measure with medical diagnostic application. Annals of Optimization Theory and Practice, 3(3), 51-66.

[33] Lin, M., Huang, C., Chen, R., Fujita, H., & Wang, X. (2021). Directional correlation coefficient measures for Pythagorean fuzzy sets: their applications to medical diagnosis and cluster analysis. Complex & Intelligent Systems, 7, 1025-1043.

[34] Thao, N. X. (2020). A new correlation coefficient of the Pythagorean fuzzy sets and its applications. Soft Computing, 24(13), 9467-9478.

[35] Mahmood, T., & Ali, Z. (2021). Entropy measure and TOPSIS method based on correlation coefficient using complex q-rung orthopair fuzzy information and its application to multi-attribute decision making. Soft Computing, 25, 1249-1275.

[36] Sarwar, M., & Li, T. (2019). Fuzzy fixed point results and applications to ordinary fuzzy differential equations in complex valued metric spaces. Hacettepe Journal of Mathematics and Statistics, 48(6), 1712-1728.

[37] Zulqarnain, R. M., Siddique, I., Jarad, F., Ali, R., & Abdeljawad, T. (2021). Development of TOPSIS Technique under Pythagorean Fuzzy Hypersoft Environment Based on Correlation Coefficient and Its Application towards the Selection of Antivirus Mask in COVID-19 Pandemic. Complexity, 2021(1), 6634991.

[38] Abid, M., & Saqlain, M. (2023). Utilizing edge cloud computing and deep learning for enhanced risk assessment in China's international trade and investment. International Journal of Knowledge and Innovation Studies, 1(1), 1-9.

[39] Peng, X., & Liu, L. (2019). Information measures for q-rung orthopair fuzzy sets. International Journal of Intelligent Systems, 34(8), 1795-1834.

[40] Xia, Y., Wang, J., Meng, B., & Chen, X. (2020). Further results on fuzzy sampled-data stabilization of chaotic nonlinear systems. Applied Mathematics and Computation, 379, 125225.

[41] Garg, H., & Arora, R. (2020). TOPSIS method based on correlation coefficient for solving decision-making problems with intuitionistic fuzzy soft set information. AIMS mathematics, 5(4), 2944-2966.

[42] Garg, H. (2016). A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. International Journal of Intelligent Systems, 31(12), 1234-1252.

[43] Ejegwa, P. A., Adah, V., & Onyeke, I. C. (2022). Some modified Pythagorean fuzzy correlation measures with application in determining some selected decision-making problems. Granular Computing, 7(2), 381-391.

[44] Abid, M., & Saqlain, M. (2023). Decision-making for the bakery product transportation using linear programming. Spectrum of Engineering and Management Sciences, 1(1), 1-12.

[45] Khan, M. J., Alcantud, J. C. R., Kumam, P., Kumam, W., & Al-Kenani, A. N. (2021). An axiomatically supported divergence measures for q-rung orthopair fuzzy sets. International Journal of Intelligent Systems, 36(10), 6133-6155.

[46] Alahmadi, R. A., Ganie, A. H., Al-Qudah, Y., Khalaf, M. M., & Ganie, A. H. (2023). Multi-attribute decision-making based on novel Fermatean fuzzy similarity measure and entropy measure. Granular Computing, 8(6), 1385-1405.

[47] Li, H., Yang, Y., & Yin, S. (2020). Two -correlation coefficients of q-rung orthopair fuzzy sets and their application to clustering analysis. Journal of Intelligent & Fuzzy Systems, 39(1), 581-591.

[48] Zhang, S., & Zhang, M. (2020). Mitigation of Bullwhip Effect in Closed-Loop Supply Chain Based on Fuzzy Robust Control Approach. Complexity, 2020(1), 1085870.

[49] Ejegwa, P. A., Onyeke, I. C., & Adah, V. (2020). An algorithm for an improved intuitionistic fuzzy correlation measure with medical diagnostic application. Annals of Optimization Theory and Practice, 3(3), 51-66.

[50] Garg, H. (2016). A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. International Journal of Intelligent Systems, 31(12), 1234-1252.

[51] Hung, W. L., & Wu, J. W. (2002). Correlation of intuitionistic fuzzy sets by centroid method. Information sciences, 144(1-4), 219-225.

[52] Ge, J., & Zhang, S. (2020). Adaptive inventory control based on fuzzy neural network under uncertain environment. Complexity, 2020(1), 6190936.

[53] Young, G. O. (1964). Synthetic structure of industrial plastics. In Plastics, 2nd edition, pp. 15-64. New York: McGraw-Hill.

[54] Zhang, N., Qi, W., Pang, G., Cheng, J., & Shi, K. (2022). Observer-based sliding mode control for fuzzy stochastic switching systems with deception attacks. Applied Mathematics and Computation, 427, 127153.

[55] Khan, M. J., Alcantud, J. C. R., Kumam, P., Kumam, W., & Al-Kenani, A. N. (2021). An axiomatically supported divergence measures for q-rung orthopair fuzzy sets. International Journal of Intelligent Systems, 36(10), 6133-6155.

[56] Lin, M., Huang, C., Chen, R., Fujita, H., & Wang, X. (2021). Directional correlation coefficient measures for Pythagorean fuzzy sets: their applications to medical diagnosis and cluster analysis. Complex & Intelligent Systems, 7, 1025-1043.

[57] Naeem, K., Riaz, M., & Afzal, D. (2019). Pythagorean m-polar fuzzy sets and TOPSIS method for the selection of advertisement mode. Journal of Intelligent & Fuzzy Systems, 37(6), 8441-8458.

[58] Riaz, M., Naeem, K., & Afzal, D. (2020). A similarity measure under pythagorean fuzzy soft environment with applications. Computational and Applied Mathematics, 39(4), 269.

[59] Hamid, M. T. & Abid, M (2024). Decision Support System for Mobile Phone Selection Utilizing Fuzzy Hypersoft Sets and Machine Learning. Journal of Intelligent Management Decision, 3(2), 104-115.

[60] Abid, M., & Shahid, M. (2024). Data-driven evaluation of background radiation safety using machine learning and statistical analysis. Big Data and Computing Visions, 4(2), 110-134.

[61] Szmidt, E., & Kacprzyk, J. (2010). Correlation of intuitionistic fuzzy sets. In International conference on information processing and management of uncertainty in knowledge-based systems (pp. 169-177). Berlin, Heidelberg: Springer Berlin Heidelberg.

[62] Duan, Z. X., Liang, J. L., & Xiang, Z. R. (2022). H control for continuous-discrete systems in TS fuzzy model with finite frequency specifications. Discrete and Continuous Dynamical Systems, 64(1), 1-18.

[63] Thao, N. X. (2020). A new correlation coefficient of the Pythagorean fuzzy sets and its applications. Soft Computing, 24(13), 9467-9478.

[64] Yager, R. R. (2013). Pythagorean fuzzy subsets. In 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS) (pp. 57-61). IEEE.

[65] Zhang, S., Li, S., Zhang, S., & Zhang, M. (2017). Decision of Lead-Time Compression and Stable Operation of Supply Chain. Complexity, 2017(1), 7436764.

[66] Yager, R. R. (2016). Generalized orthopair fuzzy sets. IEEE Transactions on Fuzzy Systems, 25(5), 1222-1230.

[67] Joshi, R. (2022). Multi-criteria decision-making based on bi-parametric exponential fuzzy information measures and weighted correlation coefficients. Granular Computing, 7, 49-62.

[68] Diao, Y., & Zhang, Q. (2021). Optimization of Management Mode of Small-and Medium-Sized Enterprises Based on Decision Tree Model. Journal of Mathematics, 2021(1), 2815086.

[69] Riaz, M., Naeem, K., & Afzal, D. (2020). Pythagorean m-polar fuzzy soft sets with TOPSIS method for MCGDM. Punjab University Journal of Mathematics, 52(3), 21-46.

[70] Gao, M., Zhang, L., Qi, W., Cao, J., Cheng, J., Kao, Y., et al. (2020). SMC for semi-Markov jump TS fuzzy systems with time delay. Applied Mathematics and Computation, 374, 125001.

[71] Gerstenkorn, T., & Mańko, J. (1991). Correlation of intuitionistic fuzzy sets. Fuzzy Sets and Systems, 44(1), 39-43.

[72] Huang, B., Miao, J., & Li, Q. (2022). A Vetoed Multi-objective Grey Target Decision Model with Application in Supplier Choice. Journal of Grey System, 34(4), 15-27.