Decision Making Advances Volume 2, Issue 1 (2024) 105-128

SCIENTIFIC OASIS

Decision Making Advances

Journal homepage: www.dma-journal.org
ISSN: 2956-2384

Multi-Attribute Decision-Making for T-Spherical Fuzzy Information Utilizing Schweizer-Sklar Prioritized Aggregation Operators for Recycled Water

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ARTICLE INFO

Article history:
Received 22 July 2023
Received in revised form 19 December 2023
Accepted 28 January 2024
Available online 15 February 2024

Keywords:
T-spherical fuzzy sets; Schweizer and Sklar norms; Hamacher norms; Lukasiewicz norms; Aggregation operators; Multi Attribute Decision-Making.

ABSTRACT

To handle problematic and ambiguous data, Schweizer and Sklar added a parameter $\overline{\boldsymbol{p}}$ in 1960, which helped to develop the theory of SS t-norm (SSTN) and t-conorm (SSTCN). The parameter $\overline{\overline{\boldsymbol{p}}} = -1.1$ can be used to easily derive the information of the Hamacher and Lukasiewicz t-norms. Furthermore, prioritized aggregation operators (PAOs) choose which data will be collected into a singleton set. The main contribution of this work is the construction of new aggregation operators for T-spherical fuzzy (T-SF) information based on SS t-norm and t-conorm. Moreover, the fundamental characteristics of the operators are identified. Further, we developed MADM (Multi Attribute Decision-Making) models and deduced several useful properties from the operators T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG. Finally, using an actual case study, we were able to draw the conclusion that, in comparison to the ground-breaking and current methods to enhance the value and capability of the diagnosed operators, the proposed MADM algorithm performs noticeably better than the operators in place for resolving the water recycling problem in a way that is easy to understand.

1. Introduction

The main purpose of MADM problems is to evaluate or identify the optimal form that maximizes the limited choice while taking into account the preference values providing by specialists or decision-makers based on the values of their attributes. Experts find it awkward and challenging to put a number on every desire in real-world issues due to the murky and intricate nature of decisions.

Zadeh [1] studied or expanded the theory of classical information in light of the previous conundrum, and developed the idea of the fuzzy set (FS). Given FS's valuable veracity and dominant

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https://doi.org/10.31181/dma21202425

shape, some have applied and demonstrated it in a variety of contexts. Mahmood and Ali [2] for example, created the fuzzy superior mandelbrot set. Orazbayeya and Plewa [3] invented the use of fuzzy logic to get results and involve the entire university administration. De *et al.,* [4] and Tang and Wang [5] investigated the qualitative proportional examination based on FS and traveled the study of strong intellect founded on FS. Comparison events for FS were industrialized by Boukezzoula *et al.,* [6]. The suggestions for exploration developments founded on fuzzy logic were distended by Cassar *et al.,* [7]. The calculation of specialists' mistake approximation founded on FS is designated by Ghasemi *et al.,* [8]. Libório *et al.,* [9] shaped an examination of the variation pointer. Deveci *et al.,* [10] showed the Delphi method for interval type-2 fuzzy evidence. Saridou *et al.,* [11] industrialized the recognition systems created on fuzzy logic. Many decision-makers have used the theory of FS in a choice of contexts, such as artificial networks, game theory, pattern recognition, and medicine. The only grade in the FS theory was the membership grade (MG), which was defined from universal set to unit interval. Modifying the non-membership grade (NMG) in the mathematical form of the FS is also essential, though. Due to the fact we came across knowledge in the form of MG and NMG in many advantageous and useful situations. In order to address this, Atanassov [12] integrated the NMG grade into the FS framework, thereby addressing the intuitionistic FS (IFS) methodology. The IF information's significant characteristic is denoted by the following: $0 \leq \bar{\bar{\mathcal{E}}}(\mathcal{X}) + \bar{\bar{\rho}}(\mathcal{X}) \leq 1$, where MG and NMG were represented by the terms $\bar{\bar{\mathcal{E}}}(\mathcal{X})$and $\bar{\bar{\rho}}(\mathcal{X})$. When managing information that cannot be provided through FSs, IF types of information are managed very practically. Furthermore, a great deal of people has applied in a lot of worthwhile fields. For example, Garg and Rani [13] created a variety of distance measurements using IF data. Garg *et al.,* [14] discussed two more measures for IF sets. Jia and Wang [15] evaluated Choquet aggregation operators using integral-based IF information, and Bal *et al.,* [16], developed the theory of subgroup in relation to IF sets. Panda and Nagwani [17] established infection preparation created on well-organized software for IF evidence, and Gu *et al.,* [18] observed the risk calculation model for IF material. Liang and Zhang [19] discussed twin SVMs based on the IF set, but Gohain *et al.,* [20] highlighted the problems with pattern recognition and clustering for the IF set. Anwar *et al.,* [21] create the training of the doubtful rough set based on the IF evidence, while Yang *et al.,* [22], well-known the three-ways ideal for the IF set. However, the sum of the MD and NMD can frequently be greater than $[0,1]$. The IFS was therefore a narrowly scoped framework. By proposing the concepts of the Pythagorean FS (PyFS) and q-rung orthopair FS (qROFS), respectively, Yager [23] and [24] extended the bounds of the IFS.

Because IFS, PYFS, and q-rungs can reduce vagueness in information extraction, their applications have a lot of potential in real-world scenarios. However, because these tools only have two degrees for an element's description, there are some scenarios in which they are unable to extract information without information loss. Cường [25] presented the picture FS (PFS) with an extra degree called abstinence degree (AD) to illustrate the belongingness of an object using three degrees. Numerous academics have used the PFS, as seen in [26]. However, occasionally, when $0 \leq \bar{\bar{\mathcal{E}}}(\mathcal{X}) + \bar{\bar{\rho}}(\mathcal{X}) \leq 1$violated, the PFS concept failed. For instance, the MD, AD, and NMD have respective values of 0.8, 0.2 and 0.4. In this case the $0 \leq \bar{\bar{\mathcal{E}}}(\mathcal{X}) + \bar{\bar{\rho}}(\mathcal{X}) \leq 1 = 1.4 \nleq 1$. Hence, the PFS was a very limited approach. As such, the PFS was an extremely constrained method. In order to expand the PFS's range, Mahmood *et al.,* [27] proposed the idea of spherical FS (SFS). Mahmood *et al.,* [27] state that T-SFSs are an extension of SFSs. There is more leeway in T-SFSs because the total of the t-n power of membership grades approaches a unit interval. Consequently, this set performs better than conventional fuzzy structures. Munir *et al.,* [28] developed the Einstein hybrid AOs for T-SFSs with applications to MCDM. A few Einstein interactive AOs for T-SFSs were defined by Zeng *et al.,* [29] using photovoltaic cell selection as an example. Liu *et al.,* [30] proposed novel power Muirhead mean

AOs for T-SFSs, while Khan *et al.,* [31] introduced the Schweizer-Sklar power Heronian mean AOs for the first time in the context of T-SFSs. Ullah *et al.,* [32] presented the idea of T-SF Hamacher AOs with MCDM. Additional research on MCDM can be found in [33-35]. Al Quran *et al.,* [36] given the concept of linear Diophantine fuzzy on the T-SFs. Garg *et al.,* [37], improved the aggregation operator on the T-SFs. Ali *et al.,* [38] Developed the concept of muti-attribute decision making on T-SFs. Liu *et al.,* [39] and Ullah *et al.,* [40] given the idea of normal T-SFs aggregation operator on muti-attribute decision making. Garg *et al.,* [41] given the concept of power aggregation operator on T-SFs, while Wang and Ullah [42] proposed uncertain linguistic MARCOS method in T-SFs environment.

The theory of Schweizer-Sklar (SS) t-norm and t-conorm was developed by Schweizer and Sklar [43]. This was achieved by adding a parameter $\overline{\overline{\boldsymbol{\rho}}}$, which is more flexible and helpful in handling complex situations. The Hamacher and Lukasiewicz t-norm information can be obtained easily by using the parameter $\overline{\overline{\boldsymbol{\rho}}} = -1.1$. Furthermore, Khan *et al.,* [44] created the IF sets SS aggregation operator, while Wang and Liu [46] derived the SS Maclaurin symmetric aggregation operators for SF sets. Furthermore, Yu and Xu [47] developed the idea of PAOs for the SF set, and Yager [48] recognized a singleton set as the collection of information. Here, the collection of data into a singleton set is also determined using PAOs. All of the previously mentioned information has been found or observed to work well in situations where boundaries are in place and decision-makers and norms are given equal weight. While some scholars have developed different types of operators but are unable to participate in the priority degree, some people have derived different types of operators based on different norms while taking priority information into consideration. As a result, they are unable to use the Schweizer-Sklar norms. Building the theory of aggregation operators based on Schweizer-Sklar t-norm and t-conorm under the assumption of priority degree is an extremely challenging task for all societies of fuzzy set theory. Because developing these ideas required combining three different kinds of frameworks, which was a difficult and uncomfortable task. These ideas can be used to easily derive the work of prioritized aggregation operators, Schweizer-Sklar aggregation operators, and basic aggregation operators, which were developed by three different researchers in the different papers. Notable and valuable are the following advantages and contributions of the developed theory and operators:

1) By contrasting the theory of T-SFS with other existing theories, such as Schweizer-Sklar aggregation operators, prioritized aggregation operators, and basic aggregation operators, they are able to more precisely and explicitly characterize the imprecise and unclear data
2) In the presence of dynamically manageable priority levels that specialists with different preferences can use, they have a considerable degree of dominance.
3) They can aggregate preferences without the use of weight vectors and correct errors brought about by biased experts' evaluation values that are either too high or too low by applying the priority degree instead of the weight vector.

Our main contribution is as follows, drawing from the discussion above for inspiration:

1) We aim to extract the meaning from the T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators, as well as reveal their useful characteristics that lead to the existence of three distinct hypotheses.
2) In order to provide some beneficial characteristics of the above-mentioned operators, we designed a MADM instrument to switch the ambiguous and untrustworthy types of information with the help of the T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators.
3) We illustrated with several concrete examples how the consequent effort is significantly better than the current operatives by comparing novel and well-established methods to increase the capacity and value of the analyzed operatives.

This paper was calculated using the ensuing profile: In Section 2 of this article, we went over specific past evidence regarding IFS and its SS working laws. We besides efficient the concept of PAOs created on optimistic numbers. In Section 3, the T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators were developed, and their useful properties and results were also covered. We pioneered the use of the T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators to handle ambiguous and unreliable types of information, and we covered several important characteristics of the aforementioned operators in Section 4. Now Section 5, we showed that the consequent effort is expressively improved than the current workers by comparing novel and well-established methods to increase the capacity and value of the analyzed operatives. We wrapped up with some enlightening observations in Section 6. The form of the derived theory is given as a geometric representation.

We derive the prioritized aggregation operator's theory of Schweizer-Sklar for T-SFSs.

1) For derived work, we derive properties of idempotency, monotonicity, and boundedness.
2) We demonstrate a multi-attribute decision-making method grounded in the approaches that have been presented.
3) We provide a numerical example that compares the suggested work to some previous research.

We talk about a few closing thoughts.

2. Methodology

In order to create some ideal and useful theories, we must update some of the current knowledge about T-SFs and their SS operational laws. Additionally, we update the theory of positive integer-based PAOs, in which the universal set is denoted by: $\overline{\overline{\mho}}$.

Definition 1[12]: It is found that a T-SFS $\overline{\overline{\emptyset}}$ in universal set $\overline{\overline{\mho}}$:

$$\overline{\overline{\emptyset}} = \left\{\left(\bar{\bar{\mathcal{E}}}^{o}(\mathcal{X}), \bar{\bar{\partial}}^{o}(\mathcal{X}), \bar{\rho}^{o}(\mathcal{X})\right) : \mathcal{X} \in \overline{\overline{\mho}}\right\}$$

In the following cases: $\bar{\bar{\mathcal{E}}}^{o}(\mathcal{X}) : \overline{\overline{\emptyset}} \to [0,1]$, $\bar{\bar{\partial}}^{o}(\mathcal{X}) : \overline{\overline{\emptyset}} \to [0,1]$, $\bar{\rho}^{o}(\mathcal{X}) : \overline{\overline{\emptyset}} \to [0,1]$ encompassing the requirement: For the element $\mathcal{X}$ in $\overline{\overline{\emptyset}}$, $0 \le \bar{\bar{\mathcal{E}}}^{o}(\mathcal{X}) + \bar{\bar{\partial}}^{o}(\mathcal{X}) + \bar{\rho}^{o}(\mathcal{X}) \le 1$. The MG, AG, and NMG of the element X in the set $\overline{\overline{\emptyset}}$ are represented by the numbers $\bar{\bar{\mathcal{E}}}^{o}(\mathcal{X})$, $\bar{\bar{\partial}}^{o}(\mathcal{X})$ and $\bar{\rho}^{o}(\mathcal{X})$, respectively. Additionally, the following is how we explained the importance of refusal information: $\overline{\overline{\delta_{\mu}}}(\mathcal{X}) = 1 - \left(\bar{\bar{\mathcal{E}}}^{o}(\mathcal{X}) + \bar{\bar{\partial}}^{o}(\mathcal{X}) + \bar{\rho}^{o}(\mathcal{X})\right)$ Lastly, the mention of the T-SF number's representation (T-SFs) is made by $\overline{\overline{\emptyset}} = \left(\overline{\overline{\mathcal{E}_{\tau}}}, \bar{\bar{\partial}}_{\tau}, \overline{\rho_{\tau}}\right), \tau = 1,2,\ldots,\alpha$.

Definition 2[24]: For any T-SFV $\overline{\overline{\emptyset_{\tau}}} = \left(\overline{\overline{\mathcal{E}_{\tau}^{o}}}, \overline{\overline{\partial_{\tau}^{o}}}, \overline{\overline{\rho_{\tau}^{o}}}\right), \tau = 1,2,\ldots,\alpha$. Then some necessary operations of the Schweizer-Sklar tools are expressed as:

$$\text{a)}\quad \overline{\overline{\emptyset_1}} \oplus \overline{\overline{\emptyset_2}} = \begin{pmatrix} \sqrt[o]{\left(1 - \left(\left(1 - \overline{\overline{\mathcal{E}_1^{o}}}\right)^{\bar{\bar{\rho}}} + \left(1 - \overline{\overline{\mathcal{E}_2^{o}}}\right)^{\bar{\bar{\rho}}}\right) - 1\right)^{\frac{1}{\bar{\bar{\rho}}}}}, \\ \sqrt[o]{\left(\left(\overline{\overline{\partial_1^{o}}}\right)^{\bar{\bar{\rho}}} + \left(\overline{\overline{\partial_2^{o}}}\right)^{\bar{\bar{\rho}}} - 1\right)^{\frac{1}{\bar{\bar{\rho}}}}}, \\ \sqrt[o]{\left(\left(\overline{\overline{\rho_1^{o}}}\right)^{\bar{\bar{\rho}}} + \left(\overline{\overline{\rho_2^{o}}}\right)^{\bar{\bar{\rho}}} - 1\right)^{\frac{1}{\bar{\bar{\rho}}}}} \end{pmatrix}$$

b) $\overline{\overline{\emptyset_1}} \otimes \overline{\overline{\emptyset_2}} = \begin{pmatrix} \sqrt[o]{\left(\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}} + \left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}} - 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(1 - \left(\left(1 - \overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}} + \left(1 - \overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}\right) - 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(1 - \left(\left(1 - \overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}} + \left(1 - \overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}\right) - 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$

c) $\Psi\overline{\overline{\emptyset_1}} = \begin{pmatrix} \sqrt[o]{\left(1 - \left(\Psi\left(1 - \overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\Psi\left(1 - \overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\Psi\left(1 - \overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$

d) $\overline{\overline{\emptyset_1}}^{\Psi} = \begin{pmatrix} \sqrt[o]{\left(\Psi\left(1 - \overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{1 - \left(\Psi\left(1 - \overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(1 - \left(\Psi\left(1 - \overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}} - (\Psi - 1)\right)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$

Definition 3 [49]: Based on defined T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}\right), \tau = 1,2, \dots, \alpha$. The accuracy function and score are derived, as follows:

$$\wedge_s\overline{\overline{\emptyset_\tau}} = \frac{\left(2 + \overline{\overline{\mathcal{E}_\tau^o}} - \overline{\overline{\partial_\tau^o}} - \overline{\overline{\rho_\tau^o}}\right)}{3}$$

$$\wedge_a\overline{\overline{\emptyset_\tau}} = \frac{\left(2 + \overline{\overline{\mathcal{E}_\tau^o}} + \overline{\overline{\partial_\tau^o}} + \overline{\overline{\rho_\tau^o}}\right)}{3}$$

Clearly, see that $\wedge_s\overline{\overline{\emptyset_\tau}} \in [-1,1]$ and $\wedge_a\overline{\overline{\emptyset_\tau}} \in [0,1]$ for all SFVs. $\overline{\overline{\emptyset_\tau}}$

To further justify the information we derive some limitations, such as we will be considered $\overline{\overline{\emptyset_1}} > \overline{\overline{\emptyset_2}}$ when we get $\wedge_s\overline{\overline{\emptyset_1}} > \wedge_s\overline{\overline{\emptyset_2}}$: we will be considered $\overline{\overline{\emptyset_1}} < \overline{\overline{\emptyset_2}}$ when we get $\wedge_s\overline{\overline{\emptyset_1}} < \wedge_s\overline{\overline{\emptyset_2}}$: but if we get $\wedge_s\overline{\overline{\emptyset_1}} = \wedge_s\overline{\overline{\emptyset_2}}$then we will follow the idea such as we will be considered $\overline{\overline{\emptyset_1}} > \overline{\overline{\emptyset_2}}$ when we get $\wedge_a\overline{\overline{\emptyset_1}} > \wedge_a\overline{\overline{\emptyset_2}}$: we will be considered $\overline{\overline{\emptyset_1}} < \overline{\overline{\emptyset_2}}$ when we get $\wedge_a\overline{\overline{\emptyset_1}} < \wedge_a\overline{\overline{\emptyset_2}}$.

Definition 4 [50]: We obtain the concept of PAOS based on the positive integers $\overline{\overline{\emptyset_\tau}} = \tau = 1,2, \dots, \alpha$ such as:

$$PA = \left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) = \mathcal{W}_1\overline{\overline{\emptyset_1}} \oplus \mathcal{W}_2\overline{\overline{\emptyset_2}} \oplus \dots . \oplus \mathcal{W}_\alpha\overline{\overline{\emptyset_\alpha}} = \oplus_{\tau=1}^{\alpha} \mathcal{W}_\tau\overline{\overline{\emptyset_\tau}}$$

Noticed that $\mathcal{W}_\tau = \frac{\mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \oplus_{\kappa}^{\tau-1} \wedge_s(\overline{\overline{\emptyset_\kappa}}), \kappa = 1, o, \dots, \alpha$.

3. Schweizer-Sklar PAOs for data on T-SF

This section's main contribution is the derivation of the T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators, as well as the discovery of their useful characteristics and outcomes.

Definition 5: For any T-SFVs $\overline{\overline{\emptyset_\tau}} = (\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}), \tau = 2,3, \dots, \alpha$, Then the notion T-SFSSPA operator is expressed as follows:

$$T - SFSSPA(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_o}}, \dots, \overline{\overline{\emptyset_\alpha}}) = \mathcal{W}_1\overline{\overline{\emptyset_1}} \oplus \mathcal{W}_2\overline{\overline{\emptyset_2}} \oplus \dots \oplus \mathcal{W}_\alpha\overline{\overline{\emptyset_\alpha}} = \oplus_{\tau=1}^{\alpha} \mathcal{W}_\tau\overline{\overline{\emptyset_\tau}} \quad (1)$$

Noticed that $\mathcal{W}_\tau = \frac{\mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \oplus_{\kappa}^{\tau-1} \wedge_s(\overline{\overline{\emptyset_\kappa}}), \kappa = 2,3, \dots, \alpha$.

Theorem 1 For any T-SFVs $\overline{\overline{\emptyset_\tau}} = (\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}), \tau = 1, 2, \dots, \alpha$. Then the integrated values of the T-SFSSPA operator still a T-SFV such as:

$$T - SFSSPA(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}) =, \begin{pmatrix} \sqrt[o]{1 - \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau (1 - \overline{\overline{\mathcal{E}_\tau^o}})^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha} \mathcal{W}_j (\overline{\overline{\partial_\tau^o}})^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha} \mathcal{W}_j (\overline{\overline{\rho_\tau^o}})^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix} \quad (2)$$

Proof: It is a very difficult task for scholars to derive Eq. (2); to do this; we employ useful and famous method of calculated instruction. Thus, in order to properly evidence $Eq. (2)$ for this, we take $\alpha = 2$ into consideration, as in:

$$\mathcal{W}_1\overline{\overline{\emptyset_1}} = \begin{pmatrix} \sqrt[o]{1 - \left(\mathcal{W}_1 (1 - \overline{\overline{\mathcal{E}_1^o}})^{\overline{\overline{\rho}}} - (\mathcal{W}_1 - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1 (\overline{\overline{\partial_1^o}})^{\overline{\overline{\rho}}} - (\mathcal{W}_1 - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1 (\overline{\overline{\rho_1^o}})^{\overline{\overline{\rho}}} - (\mathcal{W}_1 - 1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$\mathcal{W}_2\overline{\overline{\emptyset_2}} = \begin{pmatrix} \sqrt[o]{1-\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_2\left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

To further justify Eq. (2), we consider

$$T-SFSSPA\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_2}}\right) = \mathcal{W}_1\overline{\overline{\emptyset_1}} \oplus \mathcal{W}_2\overline{\overline{\emptyset_2}}$$

$$= \begin{pmatrix} \sqrt[o]{1-\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix} \oplus \begin{pmatrix} \sqrt[o]{1-\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_2\left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$
= \left(\begin{array}{c}
\sqrt[o]{\left(1-\left(\begin{array}{c}\left(1-\left(1-\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)\right)^{\overline{\overline{\rho}}}+\\ \left(1-\left(1-\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)\right)^{\overline{\overline{\rho}}}\end{array}\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\
\sqrt[o]{\left(\left(\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}+\left(\left(\mathcal{W}_2\left(\left(\overline{\overline{\partial_1^o}}\right)^{o}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\
\sqrt[o]{\left(\left(\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}+\left(\left(\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}
\end{array} \right)
$$

$$
= \left(\begin{array}{c}
\sqrt[o]{\left(1-\left(\begin{array}{c}\left(1-1+\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}+\\ \left(1-1+\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}\end{array}\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\
\sqrt[o]{\left(\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)+\left(\mathcal{W}_2\left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\
\sqrt[o]{\left(\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)+\left(\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}
\end{array} \right)
$$

$$= \begin{pmatrix} \sqrt[o]{\left(1-\left(\left(\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}+\left(\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}\right)^{\overline{\overline{\rho}}}\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\left(\overline{\overline{\partial_1^o}}\right)\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1+1+\mathcal{W}_2\left(\left(\overline{\overline{\partial_2^o}}\right)\right)^{\overline{\overline{\rho}}}-\mathcal{W}_2+1-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1+1+\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_2+1-1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[o]{\left(1-\left(\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)\right)+\left(\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_2-1)\right)\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1+1+\mathcal{W}_2\left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_2+1-1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1+1+\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_2+1-1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[o]{\left(1-\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_1-1)-(\mathcal{W}_2-1)\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(\overline{\overline{\partial_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1-\mathcal{W}_2+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1-\mathcal{W}_2+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[o]{\left(1-\left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1+1-\mathcal{W}_2+1\right)-1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1-\mathcal{W}_2+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}}+\mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}}-\mathcal{W}_1-\mathcal{W}_2+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \left(\begin{array}{c} \sqrt[o]{\left(1 - \left(\mathcal{W}_1\left(1-\overline{\overline{\mathcal{E}_1^o}}\right)^{\overline{\overline{\rho}}} + \mathcal{W}_2\left(1-\overline{\overline{\mathcal{E}_2^o}}\right)^{\overline{\overline{\rho}}} - \mathcal{W}_1 - \mathcal{W}_2 + 1\right)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}} + \mathcal{W}_2\left(\overline{\overline{\partial_1^o}}\right)^{\overline{\overline{\rho}}} - \mathcal{W}_1 - \mathcal{W}_2 + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_1\left(\overline{\overline{\rho_1^o}}\right)^{\overline{\overline{\rho}}} + \mathcal{W}_2\left(\overline{\overline{\rho_2^o}}\right)^{\overline{\overline{\rho}}} - \mathcal{W}_1 - \mathcal{W}_2 + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{array} \right)$$

$$= \left(\begin{array}{c} \sqrt[o]{1 - \left(\sum_{\tau=1}^{2} \mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{o} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{2} \mathcal{W}_\tau\left(\overline{\overline{\partial_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{o} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{2} \mathcal{W}_\tau\left(\overline{\overline{\rho_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{o} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{array} \right)$$

We supported it for $\alpha = 2$, and then we looked into it further for $\alpha = \kappa$, where it also holds true as follows:

$$= \left(\begin{array}{c} \sqrt[o]{1 - \left(\sum_{\tau=1}^{\kappa} \mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\kappa} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{\kappa} \mathcal{W}_\tau\left(\overline{\overline{\partial_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\kappa} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{\kappa} \mathcal{W}_\tau\left(\overline{\overline{\rho_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\kappa} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{array} \right)$$

Then, we prove it for $\alpha = \kappa + 1$ such as

$$T - SFSSPA\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \ldots, \overline{\overline{\emptyset_\alpha}}\right) = \mathcal{W}_1\overline{\overline{\emptyset_1}} \oplus \mathcal{W}_2\overline{\overline{\emptyset_2}} \oplus \ldots \oplus \mathcal{W}_\kappa\overline{\overline{\emptyset_\kappa}} \oplus \mathcal{W}_{\kappa+1}\overline{\overline{\emptyset_{\kappa+1}}}$$

$$= \oplus_{\tau=1}^{\kappa} \mathcal{W}_\tau\overline{\overline{\emptyset_\tau}} \oplus \mathcal{W}_{\kappa+1}\overline{\overline{\emptyset_{\kappa+1}}}$$

$$= \left(\begin{array}{c} \sqrt[o]{1 - \left(\sum_{\tau=1}^{\kappa} \mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{K} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{K} \mathcal{W}_\tau\left(\overline{\overline{\partial_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{K} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{K} \mathcal{W}_\tau\left(\overline{\overline{\rho_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{K} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{array} \right)$$

$$\oplus\left(\begin{array}{c}\sqrt[o]{1-\left(\mathcal{W}_{K+1}\left(1-\overline{\overline{\mathcal{E}_{k+1}^{o}}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_{K+1}-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\mathcal{W}_{K+1}\left(\overline{\overline{\partial_{K+1}^{o}}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_{K+1}-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\mathcal{W}_{K+1}\left(\overline{\overline{\rho_{K+1}^{o}}}\right)^{\overline{\overline{\rho}}}-(\mathcal{W}_{K+1}-1)\right)^{\frac{1}{\overline{\overline{\rho}}}}}\end{array}\right)$$

$$=\left(\begin{array}{c}\sqrt[o]{1-\left(\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}\left(1-\overline{\overline{\mathcal{E}_{\tau}^{o}}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}\left(\overline{\overline{\partial_{\tau}^{o}}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}\left(\overline{\overline{\rho_{\tau}^{o}}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\kappa+1}\mathcal{W}_{\tau}+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}\end{array}\right)$$

Here, we were successful in our evaluation of the targeted data.

We also assessed the information in Eq. (2) for idempotency, monotonicity, and boundedness.

Preposition 1: The following properties are found to hold successfully based on defined T-SFVs $\overline{\overline{\emptyset_{\tau}}}=\left(\overline{\overline{\mathcal{E}_{\tau}^{o}}},\overline{\overline{\partial_{\tau}^{o}}},\overline{\overline{\rho_{\tau}^{o}}}\right),\tau=1,2,\ldots,\alpha$

For any T-SFVs $\overline{\overline{\emptyset}}=\overline{\overline{\emptyset_{\tau}}}$ that is

$$=T-SFSSPA\left(\overline{\overline{\emptyset_{\tau_1}}},\overline{\overline{\emptyset_{\tau_2}}},\ldots,\overline{\overline{\emptyset_{\tau_\alpha}}}\right)=\overline{\overline{\emptyset}}$$

a) For any two T-SFVs $\overline{\overline{\emptyset_{\tau}}}=\left(\overline{\overline{\mathcal{E}_{\tau}}},\overline{\overline{\partial}}_{\tau},\overline{\overline{\rho}}_{\tau}\right)\leq\overline{\overline{\emptyset'_{\tau}}}=\left(\overline{\overline{\mathcal{E}'_{\tau}}},\overline{\overline{\partial}}'_{\tau},\overline{\overline{\rho}}'_{\tau}\right)$ that is $\overline{\overline{\mathcal{E}_{\tau}}}\leq\overline{\overline{\mathcal{E}'_{\tau}}}$, $\overline{\overline{\partial}}_{\tau}\geq\overline{\overline{\partial}}'_{\tau}$ and $\overline{\overline{\rho}}_{\tau}\geq\overline{\overline{\rho}}'_{\tau}$, then $T-SFSSPA\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_o}},\ldots,\overline{\overline{\emptyset_\alpha}}\right)\leq ST-FSSPA\left(\overline{\overline{\emptyset'_1}},\overline{\overline{\emptyset'_o}},\ldots,\overline{\overline{\emptyset'_\alpha}}\right)$

b) For any T-SFVs $\emptyset_{\tau}^{-}=\left\{min\overline{\overline{\mathcal{E}_{\tau}}},max\overline{\overline{\partial}}_{\tau},max\overline{\overline{\rho}}_{\tau}\right\}$ and $\emptyset_{\tau}^{+}=\left\{max\overline{\overline{\mathcal{E}_{\tau}}},min\overline{\overline{\partial}}_{\tau},min\overline{\overline{\rho}}_{\tau}\right\}$ then We have $\emptyset_{\tau}^{-}=T-SFSSPA\left(\overline{\overline{\emptyset_{\tau_1}}},\overline{\overline{\emptyset_{\tau_o}}},\ldots,\overline{\overline{\emptyset_{\tau_\alpha}}}\right)=\emptyset_{\tau}^{+}$.

Proof: These three properties were deduced in this case.

a. Accept that if $\overline{\overline{\emptyset}}=\overline{\overline{\emptyset_{\tau}}}=\left(\overline{\overline{\mathcal{E}_{\tau}^{o}}},\overline{\overline{\partial_{\tau}^{o}}},\overline{\overline{\rho_{\tau}^{o}}}\right)$. Then by using the information in Eq. (2). We have

$$T-SFSSPA\left(\overline{\overline{\emptyset_{\tau_1}}},\overline{\overline{\emptyset_{\tau_2}}},\ldots,\overline{\overline{\emptyset_{\tau_\alpha}}}\right)=$$

$$\begin{pmatrix} \sqrt[o]{1-\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(\overline{\overline{\partial^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(\overline{\overline{\rho^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[o]{1-\left(\left(1-\overline{\overline{\mathcal{E}^o}}\right)^{\overline{\overline{\rho}}}-1+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\left(\overline{\overline{\partial^o}}\right)^{\overline{\overline{\rho}}}-1+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\left(\overline{\overline{\rho^o}}\right)^{\overline{\overline{\rho}}}-1+1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}, \sum_{\tau=1}^{\alpha}\mathcal{W}_\tau = 1$$

$$= \begin{pmatrix} \sqrt[o]{1-\left(\left(1-\overline{\overline{\mathcal{E}^o}}\right)^{\overline{\overline{\rho}}}\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\left(\overline{\overline{\partial^o}}\right)^{\overline{\overline{\rho}}}\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{\left(\left(\overline{\overline{\rho^o}}\right)^{\overline{\overline{\rho}}}\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[o]{1-\left(1-\overline{\overline{\mathcal{E}^o}}\right)} \\ \sqrt[o]{\left(\overline{\overline{\partial^o}}\right)} \\ \sqrt[o]{\left(\overline{\overline{\rho^o}}\right)} \end{pmatrix} = \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho}}_\tau\right) = \overline{\overline{\emptyset}}$$

b. Accept that if

$\overline{\overline{\emptyset}}_\tau = \left(\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}\right) \le \overline{\overline{\emptyset'}}_\tau = \left(\overline{\overline{(\mathcal{E}_\tau^o)'}}, \overline{\overline{(\partial_\tau^o)'}}, \overline{\overline{(\rho_\tau^o)'}}\right)$ that is $\overline{\overline{\mathcal{E}_\tau^o}} \le \overline{\overline{(\mathcal{E}_\tau^o)'}}$, $\overline{\overline{\partial_\tau^o}} \ge \overline{\overline{(\partial_\tau^o)'}}$ and $\overline{\overline{\rho_\tau^o}} \ge \overline{\overline{(\rho_\tau^o)'}}$, then

$$\overline{\overline{\mathcal{E}_\tau^o}} \le \overline{\overline{(\mathcal{E}_\tau^o)'}} \Rightarrow 1-\overline{\overline{\mathcal{E}_\tau^o}} \le 1-\overline{\overline{(\mathcal{E}_\tau^o)'}} \Rightarrow \left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} \ge \left(1-\overline{\overline{(\mathcal{E}_\tau^o)'}}\right)^{\overline{\overline{\rho}}} \Rightarrow \sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} \ge \sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(1-\overline{\overline{(\mathcal{E}_\tau^o)'}}\right)^{\overline{\overline{\rho}}}$$

$$= \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1 \geq \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{(\mathcal{E}_\tau^o)'}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1$$

$$= \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}} \geq \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{(\mathcal{E}_\tau^o)'}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}$$

$$= \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}} \leq \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{(\mathcal{E}_\tau^o)'}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}$$

Further, we accept that $\overline{\overline{\partial_\tau}} \geq \overline{\overline{\partial_\tau'}}$, then

$\overline{\overline{\partial_\tau^o}} \geq \overline{\overline{(\partial_\tau^o)'}} \Rightarrow \overline{\overline{\partial_\tau^o}}^{\overline{\overline{\rho}}} \geq \overline{\overline{(\partial_\tau^o)'}}^{\overline{\overline{\rho}}} \Rightarrow \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\partial_\tau^o}}^{\overline{\overline{\rho}}} \geq \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\partial_\tau^o)'}}^{\overline{\overline{\rho}}} \Rightarrow \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\partial_\tau^o}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1 \geq \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\partial_\tau^o)'}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1 \Rightarrow \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\partial_\tau^o}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}} \geq \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\partial_\tau^o)'}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}$

Further, we accept that $\overline{\overline{\rho_\tau^o}} \geq \overline{\overline{(\rho_\tau^o)'}}$ then

$\overline{\overline{\rho_\tau^o}} \geq \overline{\overline{(\rho_\tau^o)'}} \Rightarrow \overline{\overline{\rho_\tau^o}}^{\overline{\overline{\rho}}} \geq \overline{\overline{(\rho_\tau^o)'}}^{\overline{\overline{\rho}}} \Rightarrow \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\rho_\tau^o}}^{\overline{\overline{\rho}}} \geq \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\rho_\tau^o)'}}^{\overline{\overline{\rho}}} \Rightarrow \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\rho_\tau^o}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1 \geq \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\rho_\tau^o)'}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1 \Rightarrow \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\rho_\tau^o}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}} \geq \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{(\rho_\tau^o)'}}^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}$

$$T - SFSSPA\left(\overline{\overline{\emptyset_{\tau_1}}}, \overline{\overline{\emptyset_{\tau_2}}}, \dots, \overline{\overline{\emptyset_{\tau_\alpha}}}\right) \leq T - SFSSPA\left(\overline{\overline{\emptyset'_{\tau_1}}}, \overline{\overline{\emptyset'_{\tau_2}}}, \dots, \overline{\overline{\emptyset'_{\tau_\alpha}}}\right)$$

C. Considering the theory in points a and point a and b we have

$$T - SFSSPA\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) \geq T - SFSSPA\left(\overline{\overline{\emptyset_1^-}}, \overline{\overline{\emptyset_2^-}}, \dots, \overline{\overline{\emptyset_\alpha^-}}\right) = \overline{\overline{\emptyset_\tau^-}}$$

$$T - SFSSPA\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) \leq T - SFSSPA\left(\overline{\overline{\emptyset_1^+}}, \overline{\overline{\emptyset_2^+}}, \dots, \overline{\overline{\emptyset_\alpha^+}}\right) = \overline{\overline{\emptyset_\tau^+}}$$

Then,

$$\overline{\overline{\emptyset_\tau^-}} \leq T - SFSSPA\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) \leq \overline{\overline{\emptyset_\tau^+}}$$

Definition 6: For any T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}\right), \tau = 1,2, \dots, \alpha$, Next, the following is how the T-SFSSWPA operator is expressed:

$$T - SFSSWPA\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) = \mathcal{W}_1 \overline{\overline{\emptyset_1}} \oplus \mathcal{W}_2 \overline{\overline{\emptyset_2}} \oplus \dots \oplus \mathcal{W}_\alpha \overline{\overline{\emptyset_\alpha}} = \oplus_{\tau=1}^{\alpha} \mathcal{W}_\tau \overline{\overline{\emptyset_\tau}}$$

Noticed that $\mathcal{W}_\tau = \frac{\varpi_\tau \mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \varpi_\tau \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \oplus_{\kappa}^{\tau-1} \wedge_s\left(\overline{\overline{\emptyset_\kappa}}\right), \kappa = 2,3, \dots, \alpha$ Additionally, the weight vector's representation is expressed as follows: $\varpi_\tau \in [0,1]$ with $\sum_{\tau=1}^{\alpha} \varpi_\tau = 1$.

Theorem 2: For any T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}_\tau^o}}, \overline{\overline{\partial_\tau^o}}, \overline{\overline{\rho_\tau^o}}\right), \tau = 1, 2, \dots, \alpha$. Then the integrated values of the T-SFSSWPA operator still a T-SFV such as:

$$T-SFSSWPA\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_2}},\ldots,\overline{\overline{\emptyset_\alpha}}\right)=,\begin{pmatrix}\sqrt[o]{1-\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}},\\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_j\left(\overline{\overline{\partial_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}},\\ \sqrt[o]{\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_j\left(\overline{\overline{\rho_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}\end{pmatrix} \tag{3}$$

Theorem 1 proof and Eq. (3) proof are equivalent universally. We also measured the evidence in Eq. (3) for idempotency, monotonicity, and boundedness.

Preposition 2: The following properties are found to hold successfully based on defined T-SFVs $\overline{\overline{\emptyset_\tau}}=\left(\overline{\overline{\mathcal{E}_\tau^o}},\overline{\overline{\partial_\tau^o}},\overline{\overline{\rho_\tau^o}}\right),\tau=1,2,\ldots,\alpha$.

a) For any T-SFVs $\overline{\overline{\emptyset}}=\overline{\overline{\emptyset_\tau}}$ that is

$$=T-SFSSWPA\left(\overline{\overline{\emptyset_{\tau_1}}},\overline{\overline{\emptyset_{\tau_2}}},\ldots,\overline{\overline{\emptyset_{\tau_\alpha}}}\right)=\overline{\overline{\emptyset}}$$

b) For any two T-SFVs $\overline{\overline{\emptyset_\tau}}=\left(\overline{\overline{\mathcal{E}_\tau^o}},\overline{\overline{\partial_\tau^o}},\overline{\overline{\rho_\tau^o}}\right)\leq\overline{\overline{\emptyset_\tau'}}=\left(\overline{\overline{(\mathcal{E}_\tau^o)'}},\overline{\overline{(\partial_\tau^o)'}},\overline{\overline{(\rho_\tau^o)'}}\right)$ that is $\overline{\overline{\mathcal{E}_\tau^o}}\leq\overline{\overline{(\mathcal{E}_\tau^o)'}}$, $\overline{\overline{\partial_\tau^o}}\geq\overline{\overline{(\partial_\tau^o)'}}$ and $\overline{\overline{\overline{\rho_\tau^o}}}\geq\overline{\overline{(\rho_\tau^o)'}}$, then $T-SFSSWPA\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_2}},\ldots,\overline{\overline{\emptyset_\alpha}}\right)\leq T-SFSSWPA\left(\overline{\overline{\emptyset_1'}},\overline{\overline{\emptyset_2'}},\ldots,\overline{\overline{\emptyset_\alpha'}}\right)$

c) For any T-SFVs $\emptyset_\tau^-=\left\{min\overline{\overline{\mathcal{E}_\tau^o}},max\overline{\overline{\partial_\tau^o}},max\overline{\overline{\rho_\tau^o}}\right\}$ and $\emptyset_\tau^+=\left\{max\overline{\overline{\mathcal{E}_\tau^o}},min\overline{\overline{\partial_\tau^o}},min\overline{\overline{\rho_\tau^o}}\right\}$ then

We have $\emptyset_\tau^-=T-SFSSWPA\left(\overline{\overline{\emptyset_{\tau_1}}},\overline{\overline{\emptyset_{\tau_2}}},\ldots,\overline{\overline{\emptyset_{\tau_\alpha}}}\right)=\emptyset_\tau^+$.

When Proposition 2 proof is comparable to Proposition 1 proof .

Definition 7: For any T-SFVs $\overline{\overline{\emptyset_\tau}}=\left(\overline{\overline{\mathcal{E}_\tau^o}},\overline{\overline{\partial_\tau^o}},\overline{\overline{\rho_\tau^o}}\right),\tau=2,3,\ldots,\alpha$, Then the notion of the T-SFSSPG operator is expressed as follows:

$$T-SFSSPG\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_2}},\ldots,\overline{\overline{\emptyset_\alpha}}\right)=\overline{\overline{\emptyset_1^{\mathcal{W}_1}}}\otimes\overline{\overline{\emptyset_2^{\mathcal{W}_2}}}\ldots\overline{\overline{\emptyset_\alpha^{\mathcal{W}_\alpha}}}=\otimes_{\tau=1}^{\alpha}\overline{\overline{\emptyset_\tau^{\mathcal{W}_\tau}}} \tag{4}$$

Noticed that $\mathcal{W}_\tau=\frac{\mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau}$, where $\mathcal{W}_1=1$ and $\mathcal{W}_\tau=\oplus_\kappa^{\tau-1}\wedge_s\left(\overline{\overline{\emptyset_\kappa}}\right),\kappa=1,2,\ldots,\alpha$.

Theorem 3: For any T-SFVs $\overline{\overline{\emptyset_\tau}}=\left(\overline{\overline{\mathcal{E}_\tau^o}},\overline{\overline{\partial_\tau^o}},\overline{\overline{\rho_\tau^o}}\right),\tau=1,2,\ldots,\alpha$. Then the integrated values of the T-SFSSPG operator still a T-SFV such as:

$$T-SFSSPG\left(\overline{\overline{\emptyset_1}},\overline{\overline{\emptyset_2}},\ldots,\overline{\overline{\emptyset_\alpha}}\right)=\begin{pmatrix}\sqrt[o]{\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_j\left(\overline{\overline{\partial_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}},\\ \sqrt[o]{1-\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_j\left(1-\overline{\overline{\rho_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}\\ \sqrt[o]{1-\left(\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau\left(1-\overline{\overline{\mathcal{E}_\tau^o}}\right)^{\overline{\overline{\rho}}}-\sum_{\tau=1}^{\alpha}\mathcal{W}_\tau+1\right)^{\frac{1}{\overline{\overline{\rho}}}}}\end{pmatrix} \tag{5}$$

Where the Theorem 1 proof and the Eq. (5) proof is comparable. We also assessed the information in Eq. (5) for idempotency, monotonicity, and boundedness.

Preposition 3: The following properties are found to hold successfully based on defined T-SFVs $\overline{\overline{\emptyset_\tau}}=\left(\overline{\overline{\mathcal{E}_\tau^o}},\overline{\overline{\partial_\tau^o}},\overline{\overline{\rho_\tau^o}}\right),\tau=1,2,\ldots,\alpha$

For any T-SFVs $\overline{\overline{\emptyset}} = \overline{\overline{\emptyset_\tau}}$ that is

$$= T - SFSSPG\left(\overline{\overline{\emptyset_{\tau_1}}}, \overline{\overline{\emptyset_{\tau_2}}}, \dots, \overline{\overline{\emptyset_{\tau_\alpha}}}\right) = \overline{\overline{\emptyset}}$$

a) For any two T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}^o_\tau}}, \overline{\overline{\partial^o_\tau}}, \overline{\overline{\rho^o_\tau}}\right) \leq \overline{\overline{\emptyset'_\tau}} = \left(\overline{\overline{(\mathcal{E}^o_\tau)'}}, \overline{\overline{(\partial^o_\tau)'}}, \overline{\overline{(\rho^o_\tau)'}}\right)$ that is $\overline{\overline{\mathcal{E}^o_\tau}} \leq \overline{\overline{(\mathcal{E}^o_\tau)'}}$, $\overline{\overline{\partial^o_\tau}} \geq \overline{\overline{(\partial^o_\tau)'}}$ and $\overline{\overline{\rho^o_\tau}} \geq \overline{\overline{(\rho^o_\tau)'}}$, then $T - SFSSPG\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) \leq T - SFSSPG\left(\overline{\overline{\emptyset'_1}}, \overline{\overline{\emptyset'_2}}, \dots, \overline{\overline{\emptyset'_\alpha}}\right)$

b) For any T-SFVs $\emptyset^-_\tau = \left\{min\overline{\overline{\mathcal{E}^o_\tau}}, max\overline{\overline{\partial^o_\tau}}, max\overline{\overline{\rho^o_\tau}}\right\}$ and $\emptyset^+_\tau = \left\{max\overline{\overline{\mathcal{E}^o_\tau}}, min\overline{\overline{\partial^o_\tau}}, min\overline{\overline{\rho^o_\tau}}\right\}$ then

$$\emptyset^-_\tau = T - SFSSPG\left(\overline{\overline{\emptyset_{\tau_1}}}, \overline{\overline{\emptyset_{\tau_2}}}, \dots, \overline{\overline{\emptyset_{\tau_\alpha}}}\right) = \emptyset^+_\tau.$$

When Proposition 3 proof is comparable to Proposition 1 proof.

Definition 8: For any T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}^o_\tau}}, \overline{\overline{\partial^o_\tau}}, \overline{\overline{\rho^o_\tau}}\right), \tau = 1,2, \dots, \alpha$, then the notion of the T-SFSSWPG operator is expressed as follow:

$$T - SFSSWPG\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) = \overline{\overline{\emptyset_1^{\mathcal{W}_1}}} \otimes \overline{\overline{\emptyset_2^{\mathcal{W}_2}}} \otimes \dots \otimes \overline{\overline{\emptyset_\alpha^{\mathcal{W}_\alpha}}} = \otimes_{\tau=1}^{\alpha} \overline{\overline{\emptyset_\tau^{\mathcal{W}_\tau}}}$$

Noticed that $\mathcal{W}_\tau = \frac{\varpi_\tau \mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \varpi_\tau \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \oplus_\kappa^{\tau-1} \wedge_s\left(\overline{\overline{\emptyset_\kappa}}\right), \kappa = 2,3, \dots, \alpha$ Additional, the illustration of the weight vector is quantified by: $\varpi_\tau \in [0,1]$ with $\sum_{\tau=1}^{\alpha} \varpi_\tau = 1$.

Theorem 4: For T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}^o_\tau}}, \overline{\overline{\partial^o_\tau}}, \overline{\overline{\rho^o_\tau}}\right), \tau = 1, 2, \dots, \alpha$. Then the integrated values of the T-SFSSWPA operator still a T-SFV such as:

$$T - SFSSWPG\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) = \begin{pmatrix} \sqrt[o]{\left(\sum_{\tau=1}^{\alpha} \mathcal{W}_j \left(\overline{\overline{\partial^o_\tau}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}}, \\ \sqrt[o]{1 - \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_j \left(1 - \overline{\overline{\rho^o_\tau}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \\ \sqrt[o]{1 - \left(\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau \left(1 - \overline{\overline{\mathcal{E}^o_\tau}}\right)^{\overline{\overline{\rho}}} - \sum_{\tau=1}^{\alpha} \mathcal{W}_\tau + 1\right)^{\frac{1}{\overline{\overline{\rho}}}}} \end{pmatrix} \quad (6)$$

Where the Theorem 1 proof and the Equation's proof are comparable. We also assessed the information in Eq. (6) for idempotency, monotonicity, and boundedness.

Preposition 4: Based on defined T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}^o_\tau}}, \overline{\overline{\partial^o_\tau}}, \overline{\overline{\rho^o_\tau}}\right), \tau = 1,2, \dots, \alpha$ we determine that the resulting properties hold efficaciously.

a) For any T-SFVs $\overline{\overline{\emptyset}} = \overline{\overline{\emptyset_\tau}}$ that is

$$= T - SFSSWPG\left(\overline{\overline{\emptyset_{\tau_1}}}, \overline{\overline{\emptyset_{\tau_2}}}, \dots, \overline{\overline{\emptyset_{\tau_\alpha}}}\right) = \overline{\overline{\emptyset}}$$

b) For any two T-SFVs $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}^o_\tau}}, \overline{\overline{\partial^o_\tau}}, \overline{\overline{\rho^o_\tau}}\right) \leq \overline{\overline{\emptyset'_\tau}} = \left(\overline{\overline{(\mathcal{E}^o_\tau)'}}, \overline{\overline{(\partial^o_\tau)'}}, \overline{\overline{(\rho^o_\tau)'}}\right)$ that is $\overline{\overline{\mathcal{E}^o_\tau}} \leq \overline{\overline{(\mathcal{E}^o_\tau)'}}$, $\overline{\overline{\partial^o_\tau}} \geq \overline{\overline{(\partial^o_\tau)'}}$ and $\overline{\overline{\rho^o_\tau}} \geq \overline{\overline{(\rho^o_\tau)'}}$, then $T - SFSSWPG\left(\overline{\overline{\emptyset_1}}, \overline{\overline{\emptyset_2}}, \dots, \overline{\overline{\emptyset_\alpha}}\right) \leq T - SFSSWPG\left(\overline{\overline{\emptyset'_1}}, \overline{\overline{\emptyset'_o}}, \dots, \overline{\overline{\emptyset'_\alpha}}\right)$

c) For any T-SFVs $\emptyset^-_\tau = \left\{min\overline{\overline{\mathcal{E}^o_\tau}}, max\overline{\overline{\partial^o_\tau}}, max\overline{\overline{\rho^o_\tau}}\right\}$ and $\emptyset^+_\tau = \left\{max\overline{\overline{\mathcal{E}^o_\tau}}, min\overline{\overline{\partial^o_\tau}}, min\overline{\overline{\rho^o_\tau}}\right\}$ then

We have $\emptyset^-_\tau = T - SFSSWPG\left(\overline{\overline{\emptyset_{\tau_1}}}, \overline{\overline{\emptyset_{\tau_2}}}, \dots, \overline{\overline{\emptyset_{\tau_\alpha}}}\right) = \emptyset^+_\tau$.

When Proposition 4's proof is comparable to Proposition1's proof

4. MADM techniques for T-SF data

Here, we want to build a MADM problem with the aid of T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators based on SF information. We did this by computing a decision matrix that contained their values as T-SF numbers.

Consider a collection of data that is communicated as a personal of alternatives $\{\beta_1, \beta_2, \ldots, \beta_m\}$ with a personal of attributes $\{\overline{\overline{\Lambda_1}}, \overline{\overline{\Lambda_2}}, \ldots, \overline{\overline{\Lambda_\alpha}}\}$. We required a weight vector $\varpi_\tau \in [0,1]$ with $\sum_{\tau=1}^{\alpha} \varpi_\tau = 1$in order to continually justify the process. Additionally, we allocate an SF quantity to every attribute in each option and attempt toward represent it in a locked medium M=$\left[\overline{\overline{\emptyset_{\iota\tau}}}\right]_{m\times\alpha}$. The calculated form of the SF quantity is given by: $\overline{\overline{\emptyset_\tau}} = \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho_\tau}}\right), \tau = 1,2, \ldots, \alpha$. Here, it is obvious that the information MG is communicated by the symbols $\overline{\overline{\mathcal{E}}}(\mathcal{X})$, AG $\overline{\overline{\partial}}(\mathcal{X})$ and NMG with the ancient and valuable characteristic: $0 \leq \left(\overline{\overline{\mathcal{E}}}^o(\mathcal{X}) + \overline{\overline{\partial}}^o(\mathcal{X}) + \overline{\overline{\rho}}^o(\mathcal{X})\right) \leq 1$. Additionally, we discussed the importance of rejection information, which is expressed as $\overline{\overline{\delta_\mu}}(\mathcal{X}) = 1 - \left(\overline{\overline{\mathcal{E}}}^o(\mathcal{X}) + \overline{\overline{\partial}}^o(\mathcal{X}) + \overline{\overline{\rho}}^o(\mathcal{X})\right)$. we demonstrated a technique whose highest assessment technique is definite lower:

Step 1: We attempt to assess the T-SF data after gathering it. If the data included cost-type values, then normalization is required. The following concepts can help with this:

$$DM = \begin{cases} \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho}}_\tau\right) for b\Lambda\alpha\Lambda\ fitkiads \\ \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho}}_\tau\right) for cost kiads \end{cases}$$

The environment does not need to be normalized when the attention data contains of assistance forms.

Step 2: We object to calculate the assessment of $\mathcal{W}_\tau = \frac{\mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \oplus_{\kappa=1}^{\tau-1} \wedge_s\left(\overline{\overline{\emptyset_\kappa}}\right), \kappa = 2,3, \ldots, \alpha$.

Step 3: In order to try and aggregate the collective information, we aim to derive the T-SF numbers based on T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators.

Step 4: Using information, we look into the accuracy and score of the data.

Step 5: Lastly, we overgrown each preference and attempt to identify the greatest advantageous ideal amongst the set of favorites.

In addition, our goal is to strengthen the value of the derived information by providing a practical example to support the above procedure.

5. Example

In this section we select the method for recycle water. Recycling water involves treating used or wastewater to remove impurities and contaminants, making it suitable for reuse. There are various methods for recycling water, and the choice of method often depends on the specific needs, the quality of the water source, and the intended use of the recycled water. Water recycling reduces the demand for freshwater sources, helping to preserve and conserve these valuable resources. As global water scarcity becomes a growing concern, recycling water becomes essential for ensuring a sustainable water supply for various uses. In regions facing water scarcity or drought conditions, water recycling provides an additional source of water, reducing reliance on traditional freshwater supplies. This is particularly important for maintaining essential services like agriculture, industry, and municipal water supply. Treating and transporting water requires energy. By recycling water locally, especially in industrial processes or agriculture, there can be energy savings compared to extracting, treating, and transporting water over long distances. Recycling water often involves treating wastewater to remove pollutants and contaminants. This helps prevent the release of harmful substances into the environment, improving water quality. Additionally, treated wastewater can be a valuable source of nutrients for agricultural use. Water recycling systems enhance resilience to drought conditions by providing alternative water sources. This is particularly important for sustaining agriculture and supporting communities during periods of water scarcity. Water recycling can lead to economic savings, especially in industries where water is a significant input. Reusing treated wastewater for non-potable purposes, such as industrial processes or landscape irrigation, can reduce operational costs. Here are some common methods:

Physical Treatment Λ_1: This involves passing water through screens or filters to remove larger particles and impurities.

Chemical Treatment Λ_2: Chemicals are added to water to form particles that can be easily removed through settling or filtration.

Biological Treatment Λ_3: This is a biological treatment method where microorganisms break down organic matter in wastewater. Natural processes in wetland environments are used to treat and purify wastewater.

Membrane Filtration Λ_4: This process uses a semipermeable membrane to remove contaminants, ions, and particles from water. A type of membrane filtration that uses slightly larger pores than reverse osmosis.

The specific method or combination of methods used depends on factors such as the quality of the source water, regulatory requirements, the intended use of the recycled water, and the available technology and infrastructure. Water recycling plays a crucial role in sustainable water management, reducing the demand on freshwater sources and mitigating environmental impacts.

The made strict evaluation for four alternatives $\Lambda_\tau(\tau = 1,2 \dots 4)$. The processes are five attributes $\beta_\tau(\tau = 1,2 \dots 4)$. $\overline{\overline{\beta_1}}$Agricultural Irrigation; $\overline{\overline{\beta_2}}$ Industrial Processes; $\overline{\overline{\beta_3}}$ Toilet Flushing; $\overline{\overline{\beta_4}}$ Groundwater Recharge and $\overline{\overline{\beta_5}}$ Construction Activities. It's important to note that the specific uses of recycled water can vary based on local regulations, treatment standards, and the quality of the recycled water. Health and safety guidelines are crucial to ensure that recycled water is treated to meet the required standards for its intended use. Public awareness and acceptance are also essential aspects of successful water recycling programs. Table 3 lists the associated T-SPFVs for linguistic terms. We will now solve the evaluation of the water reuse application using the recommended algorithm. The actions listed below ought to be taken.

Step 1: We attempt to evaluate the T-SF decision matrix by gathering examples such as Table 1. If the data included cost-type values, then normalization is required. The following concepts can help with this:

$$DM = \begin{cases} \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho}}_\tau\right) \, for benefit \; attributes \\ \left(\overline{\overline{\mathcal{E}_\tau}}, \overline{\overline{\partial}}_\tau, \overline{\overline{\rho}}_\tau\right) \, for cost \; attributes \end{cases}$$

Table 1
Alternative and Attribute

	$\overline{\overline{\boldsymbol{\Lambda_1}}}$	$\overline{\overline{\boldsymbol{\Lambda_2}}}$	$\overline{\overline{\boldsymbol{\Lambda_3}}}$	$\overline{\overline{\boldsymbol{\Lambda_4}}}$
$\boldsymbol{\beta_1}$	(0.4, 0.5,0.6)	(0.5, 0.6,0.7)	(0.3, 0.4,0.4)	(0.4, 0.3,0.5)
$\boldsymbol{\beta_2}$	(0.5, 0.4,0.3)	(0.6, 0.5,0.4)	(0.6, 0.5,0.3)	(0.4, 0.2,0.7)
$\boldsymbol{\beta_3}$	(0.6, 0.6,0.4)	(0.7, 0.7,0.5)	(0.6, 0.7,0.2)	(0.3, 0.3,0.5)
$\boldsymbol{\beta_4}$	(0.5, 0.2,0.5)	(0.6, 0.3,0.6)	(0.3, 0.6,0.3)	(0.6, 0.7,0.2)
$\boldsymbol{\beta_5}$	(0.4, 0.5,0.3)	(0.5, 0.6,0.4)	(0.3, 0.5,0.4)	(0.2, 0.3,0.7)

Step 2: Our goal is to assess the worth of $\mathcal{W}_\tau = \frac{\mathcal{W}_\tau}{\sum_{\tau=1}^{\alpha} \mathcal{W}_\tau}$, where $\mathcal{W}_1 = 1$ and $\mathcal{W}_\tau = \bigoplus_{\kappa=1}^{\tau-1} \wedge_s(\overline{\overline{\emptyset_\kappa}}), \kappa = 2,3, \dots, \alpha$
Prioritization of lternatives is presented in Tables 2 and 3.

Table 2
Contain the value of prioritization

Alternative	Score Values		$4\sum_{i=1}^{m} \overline{\overline{\Lambda_\tau}}$	$\overline{\overline{\Lambda_\tau}}$
$\boldsymbol{\beta_1}$	$\overline{\overline{\Lambda_1}} = 0.5743, \overline{\overline{\Lambda_2}} = 0.5220, \overline{\overline{\Lambda_3}} = 0.6330, \overline{\overline{\Lambda_4}} = 0.6373$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.5743, \overline{\overline{\Lambda_3}} = 0.2998, \overline{\overline{\Lambda_4}} = 0.1898$	2.0639	$\overline{\overline{\Lambda_1}} = 0.4845, \overline{\overline{\Lambda_2}} = 0.2783, \overline{\overline{\Lambda_3}} = 0.1453, \overline{\overline{\Lambda_4}} = 0.0919$
$\boldsymbol{\beta_2}$	$\overline{\overline{\Lambda_1}} = 0.6780, \overline{\overline{\Lambda_2}} = 0.6757, \overline{\overline{\Lambda_3}} = 0.6880, \overline{\overline{\Lambda_4}} = 0.5710$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6780, \overline{\overline{\Lambda_3}} = 0.4581, \overline{\overline{\Lambda_4}} = 0.3152$	2.4513	$\overline{\overline{\Lambda_1}} = 0.4080, \overline{\overline{\Lambda_2}} = 0.2766, \overline{\overline{\Lambda_3}} = 0.1869, \overline{\overline{\Lambda_4}} = 0.1286$
$\boldsymbol{\beta_3}$	$\overline{\overline{\Lambda_1}} = 0.6453, \overline{\overline{\Lambda_2}} = 0.6250, \overline{\overline{\Lambda_3}} = 0.6217, \overline{\overline{\Lambda_4}} = 0.6217$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6453, \overline{\overline{\Lambda_3}} = 0.4033, \overline{\overline{\Lambda_4}} = 0.2507$	2.2994	$\overline{\overline{\Lambda_1}} = 0.4349, \overline{\overline{\Lambda_2}} = 0.2807, \overline{\overline{\Lambda_3}} = 0.1754, \overline{\overline{\Lambda_4}} = 0.1090$
$\boldsymbol{\beta_4}$	$\overline{\overline{\Lambda_1}} = 0.6640, \overline{\overline{\Lambda_2}} = 0.6577, \overline{\overline{\Lambda_3}} = 0.5947, \overline{\overline{\Lambda_4}} = 0.6217$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6640, \overline{\overline{\Lambda_3}} = 0.4367, \overline{\overline{\Lambda_4}} = 0.2597$	2.3604	$\overline{\overline{\Lambda_1}} = 0.4237, \overline{\overline{\Lambda_2}} = 0.2813, \overline{\overline{\Lambda_3}} = 0.1850, \overline{\overline{\Lambda_4}} = 0.1100$
$\boldsymbol{\beta_5}$	$\overline{\overline{\Lambda_1}} = 0.6373, \overline{\overline{\Lambda_2}} = 0.6150, \overline{\overline{\Lambda_3}} = 0.6127, \overline{\overline{\Lambda_4}} = 0.5460$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6373, \overline{\overline{\Lambda_3}} = 0.3920, \overline{\overline{\Lambda_4}} = 0.2401$	2.2694	$\overline{\overline{\Lambda_1}} = 0.4406, \overline{\overline{\Lambda_2}} = 0.2808, \overline{\overline{\Lambda_3}} = 0.1727, \overline{\overline{\Lambda_4}} = 0.1058$

Table 3
Containe the value of prioritization weighted

Alternative	Score Values		$\boldsymbol{\varpi_\tau \overline{\overline{\Lambda_\tau}}}$	$\sum_{i=1}^{m} \boldsymbol{\varpi_\tau \overline{\overline{\Lambda_\tau}}}$	$\overline{\overline{\Lambda_\tau}}$
$\boldsymbol{\beta_1}$	$\overline{\overline{\Lambda_1}} = 0.5743, \overline{\overline{\Lambda_2}} = 0.5220, \overline{\overline{\Lambda_3}} = 0.6330, \overline{\overline{\Lambda_4}} = 0.6373$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.5743, \overline{\overline{\Lambda_3}} = 0.2998, \overline{\overline{\Lambda_4}} = 0.1898$	$\varpi_1\overline{\overline{\Lambda_1}} = 0.3000, \varpi_2\overline{\overline{\Lambda_2}} = 0.2010, \varpi_3\overline{\overline{\Lambda_3}} = 0.0300, \varpi_4\overline{\overline{\Lambda_4}} = 0.0474$	0.5784	$\overline{\overline{\Lambda_1}} = 0.5186, \overline{\overline{\Lambda_2}} = 0.3475, \overline{\overline{\Lambda_3}} = 0.0518, \overline{\overline{\Lambda_4}} = 0.0820$
$\boldsymbol{\beta_2}$	$\overline{\overline{\Lambda_1}} = 0.6780, \overline{\overline{\Lambda_2}} = 0.6757, \overline{\overline{\Lambda_3}} = 0.6880, \overline{\overline{\Lambda_4}} = 0.5710$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6780, \overline{\overline{\Lambda_3}} = 0.4581, \overline{\overline{\Lambda_4}} = 0.3152$	$\varpi_1\overline{\overline{\Lambda_1}} = 0.3000, \varpi_2\overline{\overline{\Lambda_2}} = 0.2373, \varpi_3\overline{\overline{\Lambda_3}} = 0.0458, \varpi_4\overline{\overline{\Lambda_4}} = 0.0788$	0.6619	$\overline{\overline{\Lambda_1}} = 0.4532, \overline{\overline{\Lambda_2}} = 0.3585, \overline{\overline{\Lambda_3}} = 0.0692, \overline{\overline{\Lambda_4}} = 0.1190$

Alternative	Score Values		$\boldsymbol{\varpi_\tau \overline{\overline{\Lambda_\tau}}}$	$\sum_{i=1}^{m} \boldsymbol{\varpi_\tau \overline{\overline{\Lambda_\tau}}}$	$\overline{\overline{\boldsymbol{\Lambda_\tau}}}$
$\boldsymbol{\beta_3}$	$\overline{\overline{\Lambda_1}} = 0.6453, \overline{\overline{\Lambda_2}} = 0.6250, \overline{\overline{\Lambda_3}} = 0.6217, \overline{\overline{\Lambda_4}} = 0.6217$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6453, \overline{\overline{\Lambda_3}} = 0.4033, \overline{\overline{\Lambda_4}} = 0.2507$	$\varpi_1\overline{\overline{\Lambda_1}} = 0.3000, \varpi_2\overline{\overline{\Lambda_2}} = 0.2259, \varpi_3\overline{\overline{\Lambda_3}} = 0.0403, \varpi_4\overline{\overline{\Lambda_4}} = 0.0627$	0.6289	$\overline{\overline{\Lambda_1}} = 0.4770, \overline{\overline{\Lambda_2}} = 0.3592, \overline{\overline{\Lambda_3}} = 0.0641, \overline{\overline{\Lambda_4}} = 0.0997$
$\boldsymbol{\beta_4}$	$\overline{\overline{\Lambda_1}} = 0.6640, \overline{\overline{\Lambda_2}} = 0.6577, \overline{\overline{\Lambda_3}} = 0.5947, \overline{\overline{\Lambda_4}} = 0.6217$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6640, \overline{\overline{\Lambda_3}} = 0.4367, \overline{\overline{\Lambda_4}} = 0.2597$	$\varpi_1\overline{\overline{\Lambda_1}} = 0.3000, \varpi_2\overline{\overline{\Lambda_2}} = 0.2324, \varpi_3\overline{\overline{\Lambda_3}} = 0.0437, \varpi_4\overline{\overline{\Lambda_4}} = 0.0649$	0.6410	$\overline{\overline{\Lambda_1}} = 0.4680, \overline{\overline{\Lambda_2}} = 0.3626, \overline{\overline{\Lambda_3}} = 0.0681, \overline{\overline{\Lambda_4}} = 0.1013$
$\boldsymbol{\beta_5}$	$\overline{\overline{\Lambda_1}} = 0.6373, \overline{\overline{\Lambda_2}} = 0.6150, \overline{\overline{\Lambda_3}} = 0.6127, \overline{\overline{\Lambda_4}} = 0.5460$	$\overline{\overline{\Lambda_1}} = 1, \overline{\overline{\Lambda_2}} = 0.6373, \overline{\overline{\Lambda_3}} = 0.3920, \overline{\overline{\Lambda_4}} = 0.2401$	$\varpi_1\overline{\overline{\Lambda_1}} = 0.3000, \varpi_2\overline{\overline{\Lambda_2}} = 0.2231, \varpi_3\overline{\overline{\Lambda_3}} = 0.0392, \varpi_4\overline{\overline{\Lambda_4}} = 0.0600$	0.6223	$\overline{\overline{\Lambda_1}} = 0.4821, \overline{\overline{\Lambda_2}} = 0.3585, \overline{\overline{\Lambda_3}} = 0.0630, \overline{\overline{\Lambda_4}} = 0.0965$

Step 3: We attempt to compile the collective data and derive the SF numbers based on SFSSPA, SFSSWPA, SFSSPG, and SFSSWPG operators, Table 4.

Table 4
Contained the aggregated values.

	T-SFSSPA	T-SFSSPG	T-SFSSPWA	T-SFSSPWG
$\boldsymbol{\beta_1}$	$(0.4296, 0.3511, 0.4539)$	$(0.3399, 0.5256, 0.6322)$	$(0.4425, 0.3542, 0.4848)$	$(0.3619, 0.5396, 0.6463)$
$\boldsymbol{\beta_2}$	$(0.5539, 0.2293, 0.3104)$	$(0.4551, 0.4486, 0.5184)$	$(0.5507, 0.2305, 0.3131)$	$(0.4569, 0.4455, 0.5136)$
$\boldsymbol{\beta_3}$	$(0.6374, 0.3477, 0.2246)$	$(0.3477, 0.6574, 0.4373)$	$(0.6476, 0.3498, 0.2402)$	$(0.3498, 0.6545, 0.4524)$
$\boldsymbol{\beta_4}$	$(0.5403, 0.2117, 0.2316)$	$(0.3356, 0.5342, 0.5168)$	$(0.5552, 0.2104, 0.2330)$	$(0.3588, 0.5013, 0.5371)$
$\boldsymbol{\beta_5}$	$(0.4199, 0.3483, 0.3165)$	$(0.2322, 0.5323, 0.5059)$	$(0.4363, 0.3505, 0.3147)$	$(0.2337, 0.5423, 0.4973)$

Step 4: Using information, we look into the accuracy and score of the data, Tables 5 and 6.

Table 5
Sore value representation

	T-SFSSPA	T-SFSSPG	T-SFSSPWA	T-SFSSPWG
$\boldsymbol{\beta_1}$	0.6474	0.54712	0.6428	0.5401
$\boldsymbol{\beta_2}$	0.7093	0.6215	0.7080	0.6238
$\boldsymbol{\beta_3}$	0.7352	0.5580	0.7383	0.5566
$\boldsymbol{\beta_4}$	0.7119	0.5824	0.7164	0.5884
$\boldsymbol{\beta_5}$	0.6668	0.5773	0.6696	0.5768

Table 6
Ranking order representation

Aggregation operators	Ranking and ordering
T-SFSSPA	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
T-SFSSPG	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$
T-SFSSPWA	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
T-SFSSPWG	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$

In light of score values the best solution to recycle water β_3 and β_2.

6. Influence Study

In this part, we change the value of parameters $\bar{\bar{\boldsymbol{\rho}}}$ because above fixed the value of parameters $\bar{\bar{\boldsymbol{\rho}}} = -5$. In Table 7 T-SFSSPWA result have same different parameters give the same result.

Table 7
Different parametric of T-SFSSPWA

Parametric values	Ranking and ordering
$\boldsymbol{\rho = -1}$	$\beta_4 > \beta_2 > \beta_3 > \beta_4 > \beta_5$
$\boldsymbol{\rho = -5}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -20}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -35}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -55}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -70}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -80}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -90}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -100}$	$\beta_3 > \beta_4 > \beta_2 > \beta_5 > \beta_1$

Table 8
Different parametric of T-SFSSPWG

Parametric values	Ranking and ordering
$\boldsymbol{\rho = -1}$	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$
$\boldsymbol{\rho = -5}$	$\beta_2 > \beta_3 > \beta_4 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -20}$	$\beta_2 > \beta_3 > \beta_4 > \beta_5 > \beta_1$
$\boldsymbol{\rho = -35}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$
$\boldsymbol{\rho = -55}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$
$\boldsymbol{\rho = -70}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$
$\boldsymbol{\rho = -80}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$
$\boldsymbol{\rho = -90}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$
$\boldsymbol{\rho = -100}$	$\beta_2 > \beta_3 > \beta_4 > \beta_1 > \beta_5$

In Table 8 T-SFSSPWG result have same different parameters give the same result.

7. Comparative Study

Based on the data in Table 1, we compare the pioneering operators with a few current operators in this section. As a result, we chose a number of dominant operators that were determined using some already-known data. Let's say the work produced by Khan *et al.,* [31] give the idea of SS power aggregation operator for T-SFs. Fan *et al.,* [51] problem solving COPRAS approach for multi-criteria for T-SFs and Sarfraz *et al.,* [52] give the idea of prioritized aggregation operator. The proportional investigation is exemplified in Table 9.

Table 9
Comparison information

Method	**Ranking Information**
T-SFSSPWA	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
T-SFSSPWG	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$
Khan *et al.,* [45]	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
Khan *et al.,* [45]	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$
Khan *et al.,* [31]	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
Khan *et al.,* [31]	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$
Sarfraz *et al.,* [52]	$\beta_3 > \beta_4 > \beta_3 > \beta_5 > \beta_1$
Sarfraz *et al.,* [52]	$\beta_2 > \beta_4 > \beta_5 > \beta_3 > \beta_1$

Following a thorough analysis, we found that the prevailing operators and the derived operators receive the same advantageous optimal $\beta_3$and β_2. Consequently, our deduced operators grounded in T-SFS theory represent a novel and well-established concept for handling awkward and untrustworthy data in real-world problems.

8. Conclusion

The main influence of this study is registered lower:

1) When handling information uncertainties, a T-SFs theory has been considered in which each piece of information is taken as two gradations. Additionally, we utilized the benefits of the SS TN and TCN, which are additional comprehensive than the other standard operatives since they require an supplementary parameter $\bar{\bar{\boldsymbol{\rho}}}$ and are useful for managing information.

2) The idea of the prioritized attributes is combined with the priority factor for each attribute to generate some new operators based on SS-norms. T-SFSSPA, T-SFSSWPA, T-SFSSPG, and T-SFSSWPG operators are the names of the mentioned operators. In the study, their core characteristics and traits are also underlined.

3) We provide a MADM algorithm to address the decision-making issues based on the stated operators. A numerical example is used to show how the technique is applicable.

4) By using an example and a comparison of their research with the industry standard operators, the superiority of the suggested work is shown. This study has led us to the conclusion that the projected effort is a solid solution to the MADM issues.

We will expand our decision-making algorithm in future work to accommodate various extensions of fuzzy sets, including favorite associations, data-driven agreement models, and significant decision-making issues [53-58]. Additionally, we will discuss their applicability to many fields like artificial intelligence, machine learning, game theory, etc.

Conflicts of Interest
The author declares no conflicts of interest.

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