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A Transportation Problem Considering Fixed Charge and Fuzzy Shipping Costs

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ABSTRACT

This paper considers a fixed-charge transportation problem with fuzzy shipping costs. Whereas the shipping costs of routes are fuzzy intervals, with increasing linear membership functions, fixed costs, supplies, and demands are deterministic numbers. By defining a membership function associated with the objective values and utilizing Bellman-Zadeh's max-min criterion instead of the main problem, a new crisp nonlinear mixed-integer programming problem is formulated. To solve this non-linear programming problem, at first, we find optimality conditions for a feasible solution, then, using these optimality conditions, reformulate the non-linear programming problem as a linear mixed-integer fractional programming problem, and finally transform the fractional programming problem as a mixed-integer linear programming problem, which can be solved using the existence methods. An illustrative example is solved to explain the presented details.

1. Introduction

The Fixed Charge Transportation Problem (FCTP) is an extended form of transportation problem in which a fixed cost, sometimes called a setup cost, is incurred if a distribution variable assumes a nonzero value [1-5], this problem is formulated and solved as a mixed integer network programming problem [6].

FCTP may be simply stated in terms of a distribution problem in which there are *m* suppliers (origins) and *n* customers (destinations). Each of the *m* suppliers can ship to any customer at per unit shipping cost of c_{ij} (unit cost for shipping from supplier i to customer j) plus a fixed-cost of f_{ij} assessed for opening this route. Each supplier, *i*=1,…,*m*, has S_i units of supply and each customer, *j*=1,…,*n,* demands D_j units [7-9]. The objective is to determine which routes are to be opened and

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the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized [4,10-12].

2. Problem formulation

We consider a FCTP which shipping costs are fuzzy intervals, with increasing linear membership functions; but fixed costs, supplies, demands, and decision variables are deterministic values. We assume that the shipping costs are dependent on the quality of transportation on the routes, and use a membership function to formulate this dependency, from the fuzzy logic point of view. For the simplicity of notation, let α_{ij} be the least cost of shipping through the route (*i,j*), and β_{ij} be the least cost of shipping through (*i,j*), at the highest degree of quality; it is clear that $0 < \alpha_{ij} \leq \beta_{ij}$. We define the fuzzy shipping cost $\tilde{c}_{ij} = \langle \alpha_{ij}, \beta_{ij} \rangle$, associated with the route (*i, j*), by the following membership function:

$$\boldsymbol{q}_{ij}(c_{ij}) = \begin{cases} 1 & c_{ij} \geq \beta_{ij} \\ \dfrac{c_{ij} - \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} & \alpha_{ij} \leq c_{ij} < \beta_{ij} \\ 0 & otherwise. \end{cases} \tag{1}$$

Where $\boldsymbol{q}_{ij}(c_{ij})$ shows the degree of transportation quality on route (*i, j*). Using the above notations, a FCTP with fuzzy shipping costs can be formulated as follows [3]:

$$\begin{array}{llll} \min & \tilde{Z}(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{c}_{ij} x_{ij} + f_{ij} y_{ij}) & & (a) \\ s.t. & \sum_{j=1}^{n} x_{ij} = S_i & i = 1, \dots, m & (b) \\ & \sum_{i=1}^{m} x_{ij} = D_j & j = 1, \dots, n & (c) \\ & y_{ij} = \begin{cases} 1 & if \quad x_{ij} > 0 \\ 0 & if \quad x_{ij} = 0 \end{cases} & \forall i, j & (d) \\ & x_{ij} \geq 0 & \forall i, j & (e) \\ & y_{ij} \in \{0, 1\} & \forall i, j & (f) \end{array} \tag{2}$$

Without loss of generality, we assume that: $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$. It is simple to verify that the conditional constraint (2.d) can be replaced by $x_{ij} \leq \boldsymbol{M} y_{ij}$, where *M* is a sufficiently large positive number [13-15].

The objective of the presented model is to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost is minimized and the minimum quality of transportation on routes is maximized, in fact we have two objectives in the problem. It is obvious that the objective function of the model (2) has a fuzzy structure, and minimum value of this function is not a precise value. To tackle this issue, we define a membership function associated with this objective function as follows:

$$\boldsymbol{\mu}(Z(x, y)) = \begin{cases} 1 & Z(x, y) \leq \alpha \\ \dfrac{\beta - Z(x, y)}{\beta - \alpha} & \alpha < Z(x, y) \leq \beta \\ 0 & otherwise. \end{cases} \tag{3}$$

where the constants α and β, as desirable bounds for the objective function values, are subjectively dependent on the choice of Decision-Makers. A logical and wise choice of α is to be the minimum total cost with respect to minimum shipping costs (α_{ij} s) and of β is to be the maximum total cost with respect to shipping costs at the highest quality (β_{ij} s). Now, based on the above notations, we utilize the fuzzy max-min criterion, suggested by Bellman and Zadeh [16], to reformulate the problem (2) as the following deterministic problem.

$$\max_{(\boldsymbol{x},\boldsymbol{y})\in \boldsymbol{F}} \{\min_{x_{ij}>0}\{\boldsymbol{q}_{ij}(c_{ij}), \boldsymbol{\mu}(Z(x,y))\}\} \tag{4}$$

F is the feasible region of the problem (2), and the condition $x_{ij} > 0$ has been added to (4), because there is no real expense, and no quality, if $x_{ij} > 0$. Applying the membership functions (1) and (3) we can represent (4) as the following equivalent model:

$$
\begin{array}{llll}
\max & \lambda & & (a) \\
s.t. & \lambda x_{ij} \le \dfrac{c_{ij}(\lambda)-\alpha_{ij}}{\beta_{ij}-\alpha_{ij}} x_{ij} & & (b) \\
 & \lambda \le \dfrac{\beta - \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}(\lambda)x_{ij} + f_{ij}y_{ij})}{\beta-\alpha} & & (c) \\
 & c_{ij}(\lambda)\, x_{ij} \le \beta_{ij} x_{ij} & \forall i, j & (d) \\
 & 0 \le \lambda \le 1 & & (e) \\
 & \sum_{j=1}^{n} x_{ij} = S_i & i = 1,...,m & (f) \\
 & \sum_{i=1}^{m} x_{ij} = D_j & j = 1,...,n & (g) \\
 & x_{ij} \le M y_{ij} & \forall i, j & (h) \\
 & x_{ij} \ge 0 & \forall i, j & (i) \\
 & y_{ij} \in \{0,1\} & \forall i, j & (j)
\end{array} \tag{5}
$$

where $c_{ij}(\lambda)$ denotes the minimum of the λ-cut of $\tilde{c}_{ij}$ (i.e., $\boldsymbol{q}_{ij}(c_{ij}(\lambda)) = \lambda$).

Problem (5) is a mixed-integer nonlinear programming problem, and these types of problems are normally difficult to solve. However, this problem has a special form, and we can simplify it, as a linear integer fractional problem. To this end, at first, we present an optimal condition for the solution of (5), and then reformulate this problem.

3. Optimality Conditions

Let (***x,y***)∈***F*** be a feasible solution of the model (2), we define $A(\boldsymbol{x},\boldsymbol{y}) = \{(i,j) \mid x_{ij} > 0\}$, and $d_{ij} = \beta_{ij} - c_{ij}(\lambda)$. If we fix (***x,y***) then the problem (5) is equivalent with the following linear programming problem [17,18]:

$$
\begin{aligned}
&\max \quad \lambda && && (a)\\
&s.t. \quad \lambda \le \frac{\beta_{ij} - d_{ij} - \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} && (i,j)\in A(\boldsymbol{x},\boldsymbol{y}) && (b)\\
&\lambda \le \frac{\beta - \sum_{(i,j)\in A(\boldsymbol{x},\boldsymbol{y})}((\beta_{ij} - d_{ij})x_{ij} + f_{ij})}{\beta - \alpha} && && (c)\\
&\lambda \ge 0 && && (d)\\
&d_{ij} \ge 0 && (i,j)\in A(\boldsymbol{x},\boldsymbol{y}) && (e)
\end{aligned}
\tag{6}
$$

It's obvious that decision variables in the above model are λ and d_{ij}, $(i,j)\in A(\boldsymbol{x},\boldsymbol{y})$. The following proposition presents an optimal condition for the problem (6).

Proposition 1. If (*x,y*) is a feasible solution of the model (2) then $(\lambda^{\boldsymbol{xy}}, d_{ij}^{\boldsymbol{xy}})$, $(i,j)\in A(\boldsymbol{x},\boldsymbol{y})$ is an optimal solution for (6) if:

$$
\lambda^{\boldsymbol{xy}} = \frac{\beta - \sum_{(i,j)\in A(\boldsymbol{x},\boldsymbol{y})}((\beta_{ij} - d_{ij}^{\boldsymbol{xy}})x_{ij} + f_{ij})}{\beta - \alpha} = \frac{\beta_{ij} - d_{ij}^{\boldsymbol{xy}} - \alpha_{ij}}{\beta_{ij} - \alpha_{ij}} \quad \forall (i,j)\in A(\boldsymbol{x},\boldsymbol{y})
\tag{7}
$$

4. Problem Solving

As mentioned earlier, if (*x,y*) is a fixed feasible solution of (2), Proposition 1 presents an optimality condition for the solution of the reformulated model (6), so the optimal solution of (5) is a feasible solution which its associated optimal solution in (6) has the maximum value of $\lambda^{\boldsymbol{xy}}$. However, the model (5) has many feasible solutions, and it is hard to identify the feasible solution that gives us the maximum value of $\lambda^{\boldsymbol{xy}}$. To tackle this issue, replacing $d_{ij}^{\boldsymbol{xy}}$ by $\beta_{ij} - c_{ij}(\lambda^{\boldsymbol{xy}})$ and using the optimality condition (7), we will have:

$$
c_{ij}(\lambda^{\boldsymbol{xy}}) = \alpha_{ij} + (\beta_{ij} - \alpha_{ij})\lambda^{\boldsymbol{xy}} \quad \forall (i,j)\in A(\boldsymbol{x},\boldsymbol{y})
\tag{8}
$$

and so

$$
\lambda^{\boldsymbol{xy}} = \frac{\beta - \sum_{(i,j)\in A(\boldsymbol{x},\boldsymbol{y})}(\alpha_{ij} + (\beta_{ij} - \alpha_{ij})\lambda^{\boldsymbol{xy}})x_{ij} + f_{ij})}{\beta - \alpha}
\tag{9}
$$

by solving the above equation with respect to the $\lambda^{\boldsymbol{xy}}$, we find the following explicit formula:

$$
\lambda^{\boldsymbol{xy}} = \frac{\beta - \sum_{(i,j)\in A(\boldsymbol{x},\boldsymbol{y})}(\alpha_{ij}x_{ij} + f_{ij})}{\beta - \alpha + \sum_{(i,j)\in A(\boldsymbol{x},\boldsymbol{y})}(\beta_{ij} - \alpha_{ij})x_{ij}}
\tag{10}
$$

Now, in order to find the optimal solution of (5) we can relax the feasible solution (*x,y*) and reformulate the model (5) as the following fractional programming problem

$$\begin{array}{llll}
\max & \lambda=\dfrac{\beta-\sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij}x_{ij}+f_{ij}y_{ij})}{\beta-\alpha+\sum_{i=1}^{m}\sum_{j=1}^{n}(\beta_{ij}-\alpha_{ij})x_{ij}} & & (a)\\
s.t. & \sum_{j=1}^{n}x_{ij}=S_i & i=1,...,m & (b)\\
& \sum_{i=1}^{m}x_{ij}=D_j & j=1,...,n & (c)\\
& x_{ij}\le My_{ij} & \forall i,j & (d)\\
& x_{ij}\ge 0 & \forall i,j & (e)\\
& y_{ij}\in\{0,1\} & \forall i,j & (f)
\end{array} \tag{11}$$

The above fractional model can be converted to a linear mixed integer programming problem, using the Charnes and Cooper's transformation [19-20]. Toward this end, let $\beta-\alpha+\sum_{i=1}^{m}\sum_{j=1}^{n}(\beta_{ij}-\alpha_{ij})x_{ij}=\frac{1}{\tau}$, and introduce the new variables $\hat{x}_{ij}=\tau x_{ij}, \hat{y}_{ij}=\tau y_{ij}$, the model (11) transformed to the following model:

$$\begin{array}{llll}
\max & \lambda=\beta\tau-\sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij}\hat{x}_{ij}+f_{ij}\hat{y}_{ij}) & & (a)\\
s.t. & (\beta-\alpha)\tau+\sum_{i=1}^{m}\sum_{j=1}^{n}(\beta_{ij}-\alpha_{ij})\hat{x}_{ij}=1 & & (b)\\
& \sum_{j=1}^{n}\hat{x}_{ij}=S_i\tau & i=1,...,m & (c)\\
& \sum_{i=1}^{m}\hat{x}_{ij}=D_j\tau & j=1,...,n & (d)\\
& \hat{x}_{ij}\le My_{ij} & \forall i,j & (e)\\
& \hat{y}_{ij}-\tau\ge y_{ij}-1 & \forall i,j & (f)\\
& \hat{x}_{ij}\ge 0 & \forall i,j & (g)\\
& y_{ij}\in\{0,1\} & \forall i,j & (h)\\
& \hat{y}_{ij}\ge 0 & \forall i,j & (i)
\end{array} \tag{12}$$

Constraint (12.f) forces that $\hat{y}_{ij}=\tau$ when $\hat{x}_{ij}>0$ and $\hat{y}_{ij}=0$ otherwise. Solving the model (12) by the classic methods, for large size problems, is time consuming so one can solve it using the heuristic or meta-heuristic methods. Nevertheless, in the next sub-section, we solve a small size problem, using the CPLEX solver.

Table 1
Fuzzy and fixed cost, $(\tilde{c}_{ij},f_{ij})=(\langle\alpha_{ij},\beta_{ij}\rangle,f_{ij})$, for numerical example

	Destinations					Supply
	1	2	3	4	5	
Origins	$(\langle 8,10\rangle,1)$	$(\langle 4,7\rangle,8)$	$(\langle 5,6\rangle,5)$	$(\langle 8,10\rangle,7)$	$(\langle 2,4\rangle,4)$	10
	$(\langle 3,7\rangle,5)$	$(\langle 6,9\rangle,2)$	$(\langle 4,7\rangle,5)$	$(\langle 8,9\rangle,8)$	$(\langle 3,5\rangle,6)$	20
	$(\langle 8,10\rangle,6)$	$(\langle 4,5\rangle,9)$	$(\langle 5,7\rangle,9)$	$(\langle 3,5\rangle,8)$	$(\langle 1,5\rangle,7)$	40
	$(\langle 3,6\rangle,5)$	$(\langle 5,9\rangle,8)$	$(\langle 2,5\rangle,2)$	$(\langle 7,9\rangle,9)$	$(\langle 3,4\rangle,4)$	30
Demand	20	15	15	25	25	

5. Numerical Example

Let us consider the fuzzy fixed charge transportation problem with fuzzy times and fixed costs, shown in Table 1. It is easy to check that:

$$\alpha = \min_{(x,y)\in F} \sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij}x_{ij} + f_{ij}y_{ij}) = 309, \beta = \max_{(x,y)\in F} \sum_{i=1}^{m}\sum_{j=1}^{n}(\beta_{ij}x_{ij} + f_{ij}y_{ij}) = 848$$

Based on the above data, the optimal solution of the model (12) is:

$$\begin{aligned}
&\lambda^* = 0.63956, \quad \tau^* = 0.001214,\\
&\hat{x}_{12}^* = 0.012136, \quad \hat{x}_{21}^* = 0.018204, \quad \hat{x}_{22}^* = 0.006068, \quad \hat{x}_{34}^* = 0.030340,\\
&\hat{x}_{35}^* = 0.018204 \quad \hat{x}_{41}^* = 0.006068 \quad \hat{x}_{43}^* = 0.018204, \quad \hat{x}_{45}^* = 0.012136,\\
&y_{12}^* = y_{21}^* = y_{22}^* = y_{34}^* = y_{35}^* = y_{41}^* = y_{43}^* = y_{45}^* = 1,\\
&\hat{y}_{12}^* = \hat{y}_{21}^* = \hat{y}_{22}^* = \hat{y}_{34}^* = \hat{y}_{35}^* = \hat{y}_{41}^* = \hat{y}_{43}^* = \tau^* = 0.001214.
\end{aligned}$$

all other variables are zero. Also associated optimal value of the main problem (11) is:

$$\begin{aligned}
&\lambda^* = 0.63956,\\
&x_{12}^* = \hat{x}_{12}^* / \tau^* = 10, \quad x_{21}^* = \hat{x}_{21}^* / \tau^* = 15, \quad x_{22}^* = \hat{x}_{22}^* / \tau^* = 5,\\
&x_{34}^* = \hat{x}_{34}^* / \tau^* = 25, \quad x_{35}^* = \hat{x}_{35}^* / \tau^* = 15,\\
&x_{41}^* = \hat{x}_{41}^* / \tau^* = 5, \quad x_{43}^* = \hat{x}_{43}^* / \tau^* = 15, \quad x_{45}^* = \hat{x}_{45}^* / \tau^* = 10,\\
&y_{12}^* = y_{21}^* = y_{22}^* = y_{34}^* = y_{35}^* = y_{41}^* = y_{43}^* = y_{45}^* = 1.
\end{aligned}$$

Table 2 shows the best obtained costs, using $\lambda^* = 0.63956$. The value of the objective function, with respect to the optimal solution $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ is: 80.70995.

Table 2
Optimal costs, $c_{ij}(\lambda^*) = \alpha_{ij} + (\beta_{ij} - \alpha_{ij})\lambda^*$, for numerical example

		Destinations				
		1	2	3	4	5
Origins	1	9.279126	5.918689	5.639563	9.279126	3.279126
	2	5.558252	7.918689	5.918689	8.639563	4.279126
	3	9.279126	4.639563	6.279126	4.279126	3.558252
	4	4.918689	7.558252	3.918689	8.279126	3.639563

6. Conclusions

In this paper a fixed charge transportation problem, with fuzzy shipping costs and crisp fixed costs, is formulated and solved. Since the main presented problem has a fuzzy structure, one of the most famous fuzzy decision-making criteria, i.e. Bellman and Zadeh's max-min criterion, has been used to formulate the problem as a crisp model, with nonlinear and mixed integer elements. To resolve the nonlinearity of the problem an optimal condition has been obtained and using this condition fractional programming, which is simply transformed to a mixed integer linear programming has been formulated. Although, we provide and solve a small numerical example; finding exact solutions of the problem for the large size models is a dilemma. So applying some heuristic or meta-heuristic approaches, e.g. evolutionary algorithms or artificial neural networks, to solve the presented model, can be considered as a worthwhile new research topic.

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Conflicts of Interest
The authors declare no conflicts of interest.

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